International Research Journal on Advanced Engineering and Management
https://goldncloudpublications.com
https://doi.org/10.47392/IRJAEM.2025.0391
e ISSN: 2584-2854
Volume: 03
Issue:07 July 2025
Page No: 2491 - 2496

Analytical Solution of Flat Plate Solar Collector Using Homotopy Perturbation Method

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Abstract
We present a mathematical model of rectangular cross section absorber plate solar collector whose thermal conductivity is power law functions of temperature. The energy equation of rectangular cross section absorber plate solar collector is non-linear type equation and it is solved by Homoptopy Perturbation Method. The results obtained from each equation are validated with the exact analytical solution available in the limiting condition. The effects of various thermo-physical parameters such as Solar heat flux, aspect ratio, Biot number and environmental temperature are analyzed.
Keywords: *Nonlinear Equation, Solar Collector, Power Law, Thermal Analysis.*

Nomenclature:

C	Constant that that represent dimensional less temperature
$K(t)$	Thermal conductivity function of temperature, $W/(mK)$
K_b	Thermal conductivity with respect to fluid flowing tubes, $W/(mK)$
U_l	Overall loss co-efficient, $Wm^{-2}C^{-1}$
T	Local temperature, K
P	Thermo-geometric parameter, m
T_b	Base temperature, K
T_a	Environmental temperature corresponding to k_a, K
L	Length of the absorber plate, m
x	Axial co-ordinate of the entire absorber plate, m
A	Cross-sectional area at location x, m^2
t_b	Base thickness, m
X	Dimensionless axial co-ordinate
Bi	Biot No. $\frac{U_l L}{K_b}$

Greek Symbols:

β	Parameter describe the power law of thermal conductivity, K^{-1}
θ	Dimensionless temperature,
θ_a	Non-dimensional environmental temperature of the fin corresponding to k_a,
ψ	Aspect ratio,

1. Introduction

A solar collector is a heat exchanger that converts solar radiation into thermal energy to heat a transport fluid [1]. Solar energy passes through a transparent cover and is absorbed by a high-absorptivity absorber

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plate, which transfers heat to fluid in tubes attached to the plate by welding or casting. The collector's thermal efficiency depends on heat conduction through the absorber due to incident solar radiation. Among various types, flat plate collectors are simple, easy to construct, and commonly used for low-temperature applications like drying agricultural products, wood seasoning, solar refrigeration, domestic water heating, and space heating or cooling. Heat conducts across the absorber plate perpendicular to fluid flow, allowing it to be modeled as a repeating symmetric heat transfer module, similar to conduction through a fin with insulated boundaries. Several studies on flat plate collectors assume temperature-independent thermo-physical parameters. However, temperature variations can occur, notably in materials like silicon, whose thermal conductivity varies between 300–1400 K following a power-law correlation [2]. This results in non-linear behavior in the absorber plate. Classical methods exist for solving such non-linear equations. For example, Kundu [3] used the decomposition technique to analyze thermal performance assuming linear temperature dependence of thermal conductivity and loss coefficient. Kundu and Lee [4] introduced an analytical method using separation of variables for Fourier and non-Fourier heat transfer. Variations in absorber plate thickness introduce singularities, making real-life analysis complex due to combined non-linearity and singularity.

2. Mathematical Formulation and Problem Statement

The two main important component of a collector are absorber plate and fluid flowing tubes. The analyses consider a symmetrical control volume of length 2L and non-dimensional length is measured from the symmetrical midpoint of the absorber plate. The solar energy is absorbed on the surface of absorber plate solar collector and hence the temperature of the plate increases than the environmental temperature. As the temperature of the plate is higher than the environmental temperature, heat loss is present between the plate and surroundings. This overall loss co-efficient is assumed to be constant. The thermal conductivity of the materials of the plate is assumed to be power law dependent, shown in Figure 1.

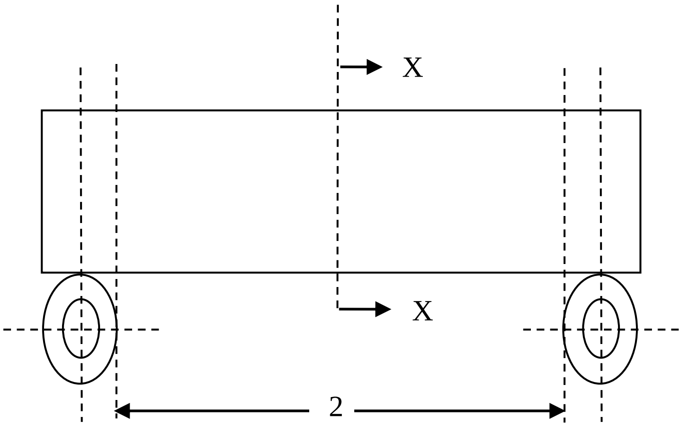

Figure 1 Schematic Diagram of Flat Plate Solar Collector

With these considerations the energy equation of the absorber plate is non-linear. Assuming 1-D and steady state and neglecting the end effect of the absorber plate, the energy equation can be written as

$$\frac{d}{dx}\left[t(x)K(T)\frac{dT}{dx}\right]=U_l\left(T-T_a\right)-S \tag{1}$$

Where thermal conductivity K (T) and thickness t(x) is defined as

$$K(t)=K_b\left(\frac{T-T_a-\frac{S}{U_l}}{T_b-T_a-\frac{S}{U_l}}\right)^{\beta} \tag{2}$$

$$t(x)=t_b\left(\frac{x}{L}\right)^{\lambda} \tag{3}$$

β is the power law of thermal conductivity and λ is the power index of non-dimensional length. The value of β is different for different materials.

Where non-dimensional terms are as follows

$$\theta=\frac{T-T_a-\frac{S}{U_l}}{T_b-T_a-\frac{S}{U_l}};\quad X=\frac{x}{L};\quad Bi=\frac{U_l t_b}{k_b};$$

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$$\psi = \frac{t_b}{L}; \quad P^2 = \frac{Bi}{\psi^2} \tag{4}$$

Eq. (1) is simplified and can be expressed in non-dimensional form as follows

$$\frac{d^2\theta}{dX^2} + \frac{\beta}{\theta}\left(\frac{d\theta}{dX}\right)^2 = P^2\left(\frac{\theta^{1-\beta}}{X}\right) \tag{5}$$

The energy equation of rectangular plate can be written as follows

$$\frac{d^2\theta}{dX^2} = P^2\theta^{1-\beta} - \beta\frac{\left(\frac{d\theta}{dX}\right)^2}{\theta} \tag{6}$$

With the following boundary conditions:

$$\frac{d\theta}{dX} = 0 \quad at \quad X = 0 \tag{7a}$$

$$\theta = 1 \quad at \quad X = 1 \tag{7b}$$

3. Homotopy Perturbation Method (HPM)

Homotopy perturbation method (HPM) [5] is a semi-numerical method for solving linear or nonlinear, homogeneous or inhomogeneous boundary value problem. This method is special case of homotopy analysis method and combines the aspects of traditional perturbation method. The advantage of HPM over the regular perturbation method eliminates the linearization or small parameter from its conventional approach. Here, the embedding parameter considered instead of small parameters provides the advantage of circumventing the linear as well as nonlinear problems. As compared to the Adomian decomposition method this method does not require the calculation Adomian polynomial and leads to convergent solution rapidly. Moreover this method requires only initial condition as input for its solution. To illustrate the basic idea of HPM according to He [6], consider the following nonlinear differential equation,

$$A(\theta) - f(r) = 0, \quad r \in \Omega. \tag{8}$$

With the boundary conditions

$$B\left(\theta, \frac{\partial\theta}{\partial X}\right) = 0 \quad r \in \Gamma, \tag{9}$$

Where A is a general differential operator, B is a boundary operator, $f(r)$ is a known analytic function, and Γ is the boundary of the domain Ω. The operator A can be generally divided into linear and nonlinear parts say $L(\theta)$ and $N(\theta)$. Therefore, the equation (4) can be written as

$$L(\theta) + N(\theta) - f(r) = 0 \tag{10}$$

Introducing the artificial parameter $p \in [0,1]$ homotopy perturbation structure of the above equation (9) as below

$$H(\theta, p) = (1-p)L(\theta - \theta_0) + p[L(\theta) + N(\theta) - f(r)] = 0. \tag{11}$$

This can be written as

$$H(\theta, p) = L(\theta) - L(\theta_0) + pL(\theta_0) + p[N(\theta) - f(r)] = 0. \tag{12}$$

Where $L = \frac{d^2}{dX^2}$ and θ_0 is the initial approximation.

Here $\theta = f(X)$

The solution of the equation (11) can be written as

$$\theta = \theta_0 + p\theta + p^2\theta_2 + p^3\theta_3 + p^4\theta_4 + \cdots\cdots \tag{13}$$

The series converges for $p = 1$ the solution for θ can be given by

$$\theta = \theta_0 + \theta + \theta_2 + \theta_3 + \theta_4 + \cdots\cdots \tag{14}$$

4. HPM Formulation

Using equation (11), the equation (5),can be written as in HPM form

$$H(\theta, p) = (1-p)L(\theta - \theta_0) + p\left[\frac{d^2\theta}{dX^2} - P^2\theta^{1-\beta} + \beta\frac{\left(\frac{d\theta}{dX}\right)^2}{\theta}\right] = 0 \tag{15}$$

The above equation can be written as

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Page No: 2491 - 2496

$$H(\theta,p)=L(\theta)-L(\theta_0)+p(\theta_0)+p\left[-P^2\theta^{1-\beta}+\beta\frac{\left(\frac{d\theta}{dX}\right)^2}{\theta}\right]=0 \quad (16)$$

Substituting θ, from equation (13), to the equation (16), and separating the variables of identical power of p

p^0 :

$$\theta_0=\theta_0 \quad (17)$$

From the boundary condition 6(a) and 6(b) it is clear that the solution is becoming meaningless. Therefore, in order to predict the solution physically meaningful the $\theta(0)$ must be a constant. This $\theta_0=C$ is taken as initial input for HPM. And p^1: This similar homotopy equations contains inhomogeneous term and as follows

$$\frac{d^2\theta_1}{dX^2}+\frac{d^2\theta_0}{dX^2}+\left[-P^2\theta^{1-\beta}-\beta\frac{\left(\frac{d\theta}{dX}\right)^2}{\theta}\right]=0 \quad (18)$$

$$\frac{d\theta_1}{dX}=0 \;\; at \;\; X=0, \quad \theta_1=0 \;\; at \;\; X=0 \quad (19)$$

And

p^2 :

$$\frac{d^2\theta_2}{dX^2}+\left[-P^2(1-\beta)\theta_0^{-\beta}\theta_1-\beta\left[\frac{\left(\frac{d\theta}{dX}\right)^2\theta_1^2}{\theta_0^2}-2\frac{\left(\frac{d\theta_0}{dX}\right)\left(\frac{d\theta_1}{dX}\right)}{\theta_0}\right]\right]=0 \quad (20)$$

By increasing number of terms in the solution higher accuracy will be obtained. Solving (18) and (20) results $\theta_1, \theta_2, and\ so\ on\cdots$

$$\theta_0=C(initial \quad approximation)$$
$$\theta_1=\frac{P^2C^{1-\beta}X^2}{2};$$
$$\theta_2=\frac{P^4(1-\beta)C^{1-\beta}X^4}{24};$$
$$\vdots \quad (21)$$

5. Results and Discussion

In this article, Homotopy perturbation method has been utilized to solve the non-linear energy equation of rectangular shape absorber plate of solar collector with power law dependent thermal conductivity. The temperature distribution of the rectangular shape absorber plate of solar collector obtained from the present semi-analytical methodology has been compared with exact analytical method available in literature in the limiting condition ($\beta=0$) and it has been observed that the present results are good agreement in as shown in Figure 2.

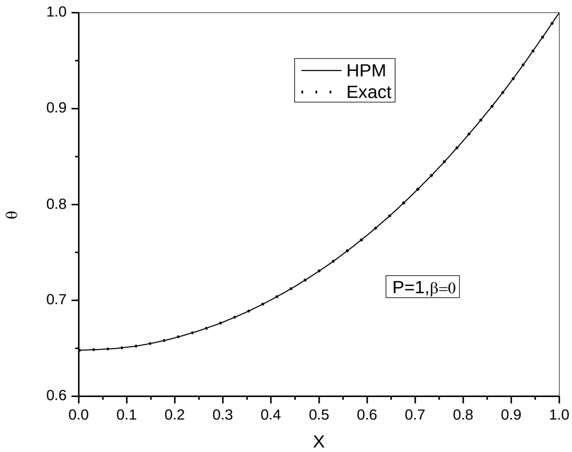

Figure 2 Comparison of Homotopy Perturbation Method (HPM) with Exact Analytical Solution in the Limiting Conditions (β=0)

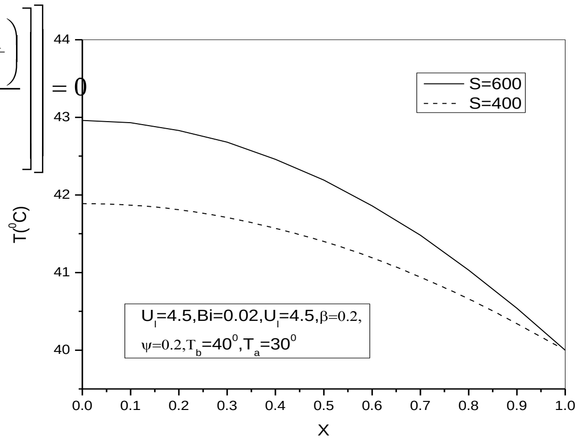

Figure 3 Effect of Solar Heat Flux on Local Temperature Distribution of Rectangular Shape Absorber Plate Solar Collector

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Page No: 2491 - 2496

Figure 3 Shows the temperature of the rectangular shape absorber plate solar collector with solar heat flux values of S=600 and S=400 respectively keeping other values at fixed level. It has been found that, the temperature of the absorber plate is higher when the value of S=600 and the temperature of the absorber plate is lower when value of S=400. This indicates that higher solar heat flux enhances the rate of heat transfer through the absorber plate solar collector. Figure 4 shows the effect of Biot number on the temperature distribution of rectangular shape absorber plate solar collector. The higher value of Biot Number, higher is the plate temperature of rectangular shape absorber plate solar collector. This indicates that Biot number is a pertinent design parameter that effects the thermal characteristics of solar collector.

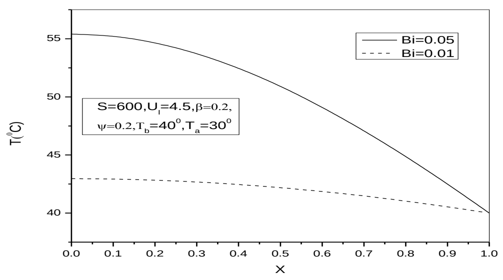

Figure 4 Effect of Biot Number on Dimensional Temperature of Rectangular Shape Absorber Plate Solar Collector

Figure 5 shows the effect of aspect ratio on the temperature distribution of rectangular shape absorber plate solar collector. The aspect ratio is the important design parameter that provides the relationship between the thickness and length of the absorber plate. For lower values of aspect ratio, higher is the tip temperature of the absorber plate solar collector and vice-versa. Thus in order to practically design a solar collector the consideration of aspect ratio is an important design parameter for thermal aspect. Figure 6 shows the environmental temperature on the temperature distribution of rectangular plate solar collector. The higher the environmental temperature has little effect on the temperature distribution of absorber plate solar collector as compared to the other thermophysical parameter.

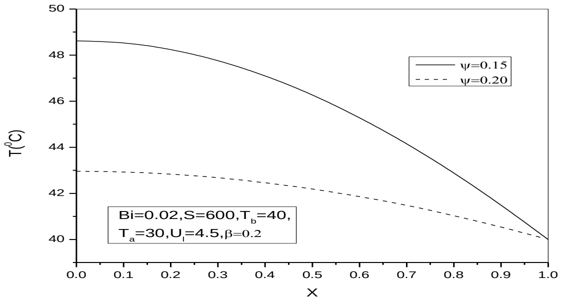

Figure 5. Effect of Aspect Ratio on Local Temperature Distribution

Figure 5 shows the effect of Effect of Aspect Ratio on Local Temperature Distribution of Rectangular Shape Absorber Plate Solar Collector While Other Parameter at Fixed Level. It Has Been Found That with The Increase of Environmental Temperature 30 To 40 Leads to An Increase of Absorber Plate Temperature of The Solar Collector.

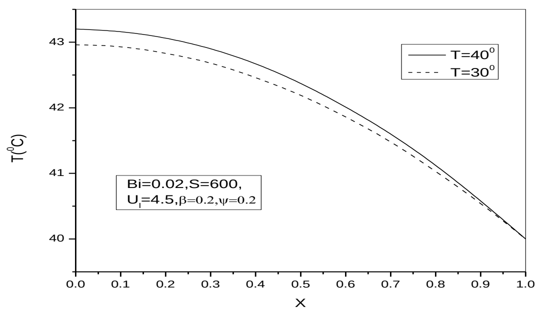

Figure 6 Effect of Environmental on Dimensional Temperature of Rectangular Shape Absorber Plate Solar Collector

Conclusions

A semi-analytical solution of rectangular shape solar plate solar collector with is power law variable of thermal conductivity has been analyzed. The complex mathematical equation of the rectangular shape absorber plate solar collector has been solved by homotopy perturbation method. The results obtained from the equation are validated separately with the exact analytical solution. The effects of various thermo-physical parameters such as solar heat flux, aspect ratio, Biot number and environmental temperature are analyzed.

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Page No: 2491 - 2496

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