

Application of Fermatean Vague Soft Sets in Multi-Criteria Decision-Making for Cargo UAV Selection

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Abstract

Rapid technological advancements have significantly streamlined logistics and transport systems, with Unmanned Aerial Vehicles (UAVs) emerging as a practical alternative in regions where conventional transport infrastructure is limited or ineffective. However, selecting an optimal cargo drone requires balancing complex, often competing criteria across operational and financial factors. To address this, we apply a Fermatean Vague Soft Set (FVSS) framework within a multi-criteria decision-making (MCDM) process to identify the most suitable cargo UAV. By integrating the parameterization of soft sets with the nuanced truth and falsity bounds of vague sets, the FVSS model expands the operational space via the cubic constraint, providing a flexible and reliable tool to handle uncertain, incomplete, and ambiguous expert evaluations.

Keywords: Fermatean vague soft set; MCDM; UAV; Vague soft set

1. Introduction

Modern logistical supply chains face increasing demands for rapid, cost-effective, and adaptable point-to-point delivery services. In geographical terrains where conventional ground infrastructure—such as roadways, bridges, and rail networks—is either lacking, damaged by natural disasters, or economically impractical to construct, Unmanned Aerial Vehicles (UAVs) have emerged as an effective freight delivery solution. From middle-mile transport to emergency medical distribution, autonomous cargo drones offer unique operational agility. However, the acquisition and fleet integration of industrial-grade cargo UAVs involves balancing competing performance metrics. Aircraft with exceptional endurance and high payload capacities frequently command substantial capital costs, whereas economical platforms may lack the advanced sensor fusion, weather resilience, and communication ranges required for autonomous flight in unsegregated airspace. Evaluating these multi-attribute trade-offs constitutes a discrete Multi-Criteria Decision-Making (MCDM) problem characterized by substantial degrees of uncertainty, incomplete

operational data, and subjective expert evaluations. To handle ambiguous and non-linear human reasoning, Zadeh introduced fuzzy set theory[1], which was subsequently extended by Atanassov to intuitionistic fuzzy sets (IFS)[2] to model membership, non-membership, and hesitation. Vague sets, as formulated by Gau and Buehrer[3], further refined these concepts by providing lower and upper boundaries on truth values. While Molodtsov's soft set theory [4] introduced parameterized families of subsets to eliminate parameterization inadequacies, combining vague sets with soft sets yielded Vague Soft Sets (VSS)[5]. Recently, Al-shboul et al. established the concept of the Fermatean Vague Soft Set (FVSS)[7], integrating the cubic Fermatean constraint ($m^3 + n^3 \leq 1$) [6] with vague soft sets. This framework grants an expanded degree of freedom over intuitionistic and Pythagorean limits, making it well-suited for high-dimensional engineering evaluations. In this paper, we develop an MCDM model based on the FVSS framework specifically tailored to evaluate and select cargo UAV platforms. By mapping continuous

engineering specifications and expert qualitative appraisals onto a parameterized Fermatean linguistic scale, we derive a transparent, reproducible selection algorithm.

2. Preliminaries

2.1. Soft Sets and Vague Soft Sets

Let U be an initial universal set of objects and E be a set of parameters or attributes related to $P(U)$. Let $P(U)$ denote the power set of U and $A \subseteq E$.

Definition 1 (Soft Set): A pair (F, A) is called a soft set over U , where F is a mapping given by $F: A \rightarrow P(U)$. [4]

Definition 2 (Vague Soft Set): Let $V(U)$ denote the collection of all vague subsets of U . A pair $(\hat{F}, A)$ is called a vague soft set over U , where $\hat{F}$ is a mapping defined by $\hat{F}: A \rightarrow V(U)$. For any parameter $e \in A$, $\hat{F}(e)$ is a vague set characterized by a truth-membership function $t_{\hat{F}(e)}(u)$ and a false-membership function $f_{\hat{F}(e)}(u)$, where function $t_{\hat{F}(e)}(u) \leq 1 - f_{\hat{F}(e)}(u)$, for all $u \in U$. [5]

2.2. Fermatean Vague Soft Set (FVSS)

Building upon the work of Al-shboul et al.: [7]

Definition 3 (Fermatean Vague Soft Set): Let U be a universe of discourse and E be a set of parameters, with $A \subseteq E$. A Fermatean Vague Soft Set (FVSS) over (U, E) , denoted by $\tilde{F}_{\tilde{A}}$, is characterized by a mapping $\tilde{F}: A \rightarrow FV(U)$, where $FV(U)$ denotes the set of all Fermatean vague subsets of U . For each parameter $e_i \in A$ and each object $u_j \in U$, the evaluation is defined as:

$$\tilde{F}_{(e_i)} = \left\{ \left(u_j, \langle m_{u_j}(e_i), n_{u_j}(e_i), h_{u_j}(e_i) \rangle \right) : u_j \in U \right\}$$

where $m_{u_j}(e_i) \in [0, 1]$ represents the degree of truth-membership, $n_{u_j} \in [0, 1]$ represents the degree of false-membership (or non-membership), subject to the cubic Fermatean condition:

$$m_{u_j}(e_i)^3 + n_{u_j}(e_i)^3 \leq 1.$$

The hesitation margin, $h_{u_j}(e_i)$ is determined as:

$$h_{u_j}(e_i) = 1 - m_{u_j}(e_i)^3 - n_{u_j}(e_i)^3$$

This cubic formulation accommodates assessments where $m + n > 1$ and $m^2 + n^2 > 1$ permitting wider flexibility in capturing expert hesitation without discarding valid evaluative boundaries. [6,7].

3. Decision-Making Methodology Under Fvss

To solve the multi-attribute selection problem, we implement the following operational procedure:

Step 1: Formulation of Alternatives and Criteria

Let $U = \{u_1, u_2, \dots, u_m\}$ be the universe of candidate alternatives and $E = \{e_1, e_2, \dots, e_k\}$ be the set of operational evaluation criteria. The criteria are partitioned into beneficial criteria (where higher values are preferred) and non-beneficial/cost criteria (where lower values are preferred).

Step 2: Linguistic Mapping and Evaluation Matrix Construction

Decision-makers evaluate the alternatives under each parameter using a calibrated linguistic scale. Non-beneficial criteria are inverted such that superior economic performance (lower absolute cost) corresponds to higher truth-membership. Each qualitative rating is converted into its corresponding Fermatean vague triplet $\langle m, n, h \rangle$ [7] satisfying the cubic condition.

Step 3: Parameter Score Function Calculation

For each alternative u_j under parameter e_i the net score $S_{e_i}(u_j)$ is computed via the difference between its membership and non-membership values:

$$S_{e_i}(u_j) = m_{u_j}(e_i) - n_{u_j}(e_i) \quad [7]$$

Step 4: Aggregate Scoring and Final Ranking

The total performance score $S(u_j)$ for each alternative across all k parameters is obtained by computing the cumulative sum:

$$S(u_j) = \sum_{i=1}^k S_{e_i}(u_j) \quad [7]$$

The optimal candidate u^* is selected according to the maximal score criterion:

$$u^* = \operatorname{argmax}_{u_j} S(u_j) [7]$$

4. Case Study: Cargo Uav Selection

4.1.Problem Description

A logistics enterprise intends to procure a fleet of unmanned aerial systems to establish middle-mile supply lines over mountainous terrain. Four distinct cargo drone platforms are identified as candidate alternatives:

u_1 : Features triple-redundant autopilot, GPS-free sensor fusion, and Active Electronically Scanned Array (AESA) radar.

u_2 : Outfitted with pneumatic de-icing boots and triple-redundant avionics architectures.

u_3 : Equipped with Differential GPS (DGPS) and automated flight trajectory management with dynamic rerouting.

u_4 : Integrated with Synthetic Aperture Radar (SAR), Ground Moving Target Indication (GMTI), and Electro-Optical/Infrared (EO/IR) sensors. The enterprise evaluates these platforms across five operational criteria:

The enterprise evaluates these platforms across five operational criteria:

e_1 (Acquisition Cost): Total procurement expenditure in millions of USD (non-beneficial parameter: lower is better).

e_2 (Payload Capacity): Maximum cargo lifting capability in kilograms (beneficial parameter).

e_3 (Flight Endurance): Maximum sustained airborne duration in hours (beneficial parameter).

e_4 (Communication Range): Maximum operational command-and-control data link radius in kilometers (beneficial parameter).

e_5 (Sensor and Avionics System): Technological maturity, autonomous fail-safe capabilities, and navigation robustness (beneficial parameter).

The baseline technical specifications for the four candidate UAVs are summarized in Table 1. Performance data for selected UAV alternatives (adapted from Yalçın & Arslan, 2026)[8].

Table 1 Baseline Specifications of Candidate Cargo UAVs

Parameter / Criterion	Drone u_1	Drone u_2	Drone u_3	Drone u_4
e_1:Cost (\$ Millions)	\$5.37	\$5.00	\$2.00	\$5.00
e_2:Payload (kg)	150	190	150	150
e_3:Endurance (hours)	24	20	20	17
e_4:Range(km)	150	200	200	200
e_5:Sensors/ Avionics	Ultra-Advanced	Advanced	Very Good	Advanced

4.2.Linguistic Evaluation Scale

To convert both technical data and qualitative expert judgments into mathematical structures, a seven-level Fermatean linguistic scale is employed, as detailed in Table 2. For the cost criterion (e_1), a min-max inverse normalization approach ((max-value)/(max-min)) is utilized to align lower procurement prices with higher satisfaction membership.[8]

Table 2 Fermatean Linguistic Evaluation Scale

Linguistic Grade	Membership (m)	Non-membership (n)	Calculated Hesitation ($h=1-m^3-n^3$)
Absolutely Excellent / Best	0.95	0.10	$1-(0.95)^3-(0.10)^3=0.1416$
Very High	0.85	0.25	$1-(0.85)^3-$

/ Very Good			$(0.25)^3$ =0.3699
High / Good	0.75	0.35	$1-(0.75)^3-$ $(0.35)^3$ =0.5346
Medium / Fair	0.50	0.50	$1-(0.50)^3-$ $(0.50)^3$ =0.7500
Low / Poor	0.35	0.75	$1-(0.35)^3-$ $(0.75)^3$ =0.5346
Very Low / Very Poor	0.25	0.85	$1-(0.25)^3-$ $(0.85)^3$ =0.3699
Absolutely Terrible / Worst	0.10	0.95	$1-(0.10)^3-$ $(0.95)^3$ =0.1416

4.3.Parametric FVSS Representation

Applying the scale to the candidate models yields the parameterized FVSS sets across the universe

$$U = \{u_1, u_2, u_3, u_4\}:$$

Parameter e_1 (Cost):

$$\tilde{F}(e_1)=\left\{\frac{u_1}{\langle 0.35,0.75,0.5346 \rangle}, \frac{u_2}{\langle 0.50,0.50,0.7500 \rangle}, \frac{u_3}{\langle 0.85,0.25,0.3699 \rangle}, \frac{u_4}{\langle 0.50,0.50,0.7500 \rangle}\right\}$$

Parameter e_2 (Payload):

$$\tilde{F}(e_2)=\left\{\frac{u_1}{\langle 0.35,0.75,0.5346 \rangle}, \frac{u_2}{\langle 0.95,0.10,0.1416 \rangle}, \frac{u_3}{\langle 0.35,0.75,0.5346 \rangle}, \frac{u_4}{\langle 0.35,0.75,0.5346 \rangle}\right\}$$

Parameter e_3 (Endurance):

$$\tilde{F}(e_3)=\left\{\frac{u_1}{\langle 0.95,0.10,0.1416 \rangle}, \frac{u_2}{\langle 0.75,0.35,0.5346 \rangle}, \frac{u_3}{\langle 0.75,0.35,0.5346 \rangle}, \frac{u_4}{\langle 0.50,0.50,0.7500 \rangle}\right\}$$

• **Parameter e_4 (Range):**

$$\tilde{F}(e_4)=\left\{\frac{u_1}{\langle 0.50,0.50,0.7500 \rangle}, \frac{u_2}{\langle 0.85,0.25,0.3699 \rangle}, \frac{u_3}{\langle 0.85,0.25,0.3699 \rangle}, \frac{u_4}{\langle 0.85,0.25,0.3699 \rangle}\right\}$$

• **Parameter e_5 (Sensors):**

$$\tilde{F}(e_5)=\left\{\frac{u_1}{\langle 0.95,0.10,0.1416 \rangle}, \frac{u_2}{\langle 0.50,0.50,0.7500 \rangle}, \frac{u_3}{\langle 0.75,0.35,0.5346 \rangle}, \frac{u_4}{\langle 0.85,0.25,0.3699 \rangle}\right\}$$

4.4.Results and Discussion

Using the score function $S_{e_i}(u_j) = m_{u_j}(e_i) - n_{u_j}(e_i)$, individual parametric scores are computed for each candidate. Table 3 details the score breakdown and the final aggregate totals across all five criteria.

Table 3 Comprehensive Decision Evaluation and Scoring Matrix

Drone s	e_1 (Cost)	e_2 (Payload)	e_3 (Endurance)	e_4 (Range)	e_5 (Sensors)	Aggregate Score $S(u_j)$
u1	-0.40	+0.40	+0.85	0.00	+0.85	+1.70
u2	0.00	+0.85	+0.40	+0.60	0.00	+1.85
u3	+0.60	+0.40	+0.40	+0.60	+0.40	+2.40
u4	0.00	+0.40	0.00	+0.60	+0.60	+1.60

From the computed cumulative scores, the final priority ranking of the four candidate UAVs is established as:

$$u_3 > u_2 > u_1 > u_4$$

4.5.Operational Interpretation

- Drone **u3** (Rank 1): Achieves the highest score (S=2.40), establishing it as the optimal cargo delivery UAV. While **u3**

does not lead in standalone endurance or sensor sophistication, its low procurement cost (\$2.00M, translating to a +0.60 net score), combined with dependable communication range (200 km) and reliable automated rerouting avionics, yields the most balanced operational profile.

- Drone **u2** (Rank 2): Ranks second ($S=1.85$) primarily due to its 190 kg payload capacity (+0.85 net score). However, a moderate unit cost (\$5.00M) and baseline sensor scores limit its overall standing.
- Drone **u1** (Rank 3): Exhibits exceptional endurance (24 hours) and advanced AESA radar. Nevertheless, its high procurement cost (\$5.37M) produces an ei penalty of -0.40 , which, combined with a shorter 150 km communication range, drops its aggregate score to 1.70.
- Drone **u4** (Rank 4): Attains a total score of 1.60. Despite featuring SAR and GMTI surveillance payloads, its lower flight endurance (17 hours) and standard cost produce the lowest cumulative rating.

Conclusion And Future Directions

This study addressed the multi-attribute selection of cargo UAVs by implementing a decision-making framework based on Fermatean Vague Soft Sets (FVSS). By operating under the relaxed cubic constraint $m3+n3 \leq 1$, the model accommodated complex trade-offs between physical flight parameters and financial limitations without losing evaluative nuances to arbitrary threshold cutoffs. The empirical findings established drone **u3** as the optimal platform, providing a balanced trade-off between acquisition cost, payload, and operational reliability. Future extensions of this work include integrating objective criterion weighting techniques (such as Entropy-weight methods or analytic hierarchy processes) to remove uniform parameter assumptions. Furthermore, incorporating the mathematical structure of FVSS into dynamic, real-time control algorithms—such as observer-based sliding mode control for stochastic UAV trajectory planning in turbulent airspaces—represents a promising research

trajectory.

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