



Bending Analysis of Smart Functionally Graded CNT Reinforced Composite Plates

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Abstract

Analysis of smart functionally graded plate of carbon nanotube reinforced composites combined with piezoelectric constituents has been introduced by finite element model (FEM). Material parameters of CNT reinforced composite plates are supposed to vary along the thickness direction following extended rule of mixture. Finite element model of the FG plates combined with piezoelectric materials is developed using ANSYS software. This model is tested against previously published work and found to be in accord with them. The numerical results are evaluated using verified boundary conditions. The effectiveness of piezoelectric materials for control of deflection of this FG plate is presented. The results revealed that the piezoelectric layer of the smart FG plates efficiently controls the nonlinear deformations of the presently studied smart CNT reinforced functionally graded plate. Results are also presented considering different material profiles for the grading of the FG plate.

Keywords: Finite Element Method; Carbon Nanotubes; Volume Fraction; Functionally Graded Materials.

1. Introduction

The functionally graded CNT reinforced composite plate structure is an essential structure element frequently used in leading civil as well as aerospace structures. The static and dynamic characteristic of these functionally graded plate structures is different than those of functionally graded beams. Therefore, the detailed analysis of functionally graded CNTRC plate becomes necessary. In this part presents a study of smart functionally graded (FG) plates made of carbon nanotube (CNT) reinforced composites. The composite plate's material parameters are assumed to vary along its thickness according to the extended rule of mixture. A simulation model for the smart FGCNTRC plate is developed in ANSYS software. Quian et al [1] worked on the load transfer and deformation mechanism of CNT reinforced composite. It was observed that the adding of small amount CNT resulted in increase in break stress and elastic modulus. Shen [2] was the first to suggest CNT based FG material with different distribution within an isotropic matrix. FGCNTRC plate show a piece wise continuous displacement field in the thickness

direction. Kumar and Sarangi [3] study on FGCNTRC beam by finite element method and observed that volume fraction of CNT is significantly affect the bending deflection. Zhu & Alibeigloo et al [4,5] work on static and free vibration of FGCNTRC plate. They observed that the CNT volume fraction and width to thickness ratio has critically effect on FGCNTRC plate. Reddy and chegg [6] work on the three-dimensional smart FG plate. It is discovered that the volume fraction distribution is only significant in relation to the applied temperature field. Ray and sachade [8] work on the smart FG plate using FE method and observed that piezoelectric fibre gives maximum performance if it is integrated with the softest surface of FG plate. Rouzegar and abbasi [9] study on the bending analysis of smart FG plate. They observed that piezoelectric fiber reinforced composite (PFRC) actuator is less effective in case of thin substrate than for thick substrate. Kumar and Sarangi [10] work on the free vibration analysis of smart FGCNTRC beam and observed that natural frequency of beam depend on the boundary

condition aspect ratio and volume fraction of CNT. Alghanmi & Jin et al [11,12] study on the FGCNTRC plate with piezoelectric actuator and investigate that the maximum value of transverse shear stress is obtained at the interference between PFRC and FGCNTRC laminated when electric voltage is applied. Haydarpour et al [13] work on free vibration analysis of FGCNTRC truncated conical shell. They observed that the fundamental frequency depends on the semi vertex angle of FGCNTRC shell and angular velocity. Zhang et al [14] work on the FGCNTRC triangular plate using FSDT and Ritz method. They conclude that non dimensional frequency of triangular plate is minimum for SSS boundary condition. Alibeigloo [15,16] work on the FGCNTRC cylinder panel integrated with piezoelectric layer using theory of elasticity. They conclude that influence of CNT volume fraction is more effect on thin panel than thick panel. In present research, Smart FGCNT reinforced composite plate combined with Piezoelectric materials is analysed using FEM Method. The material characteristics for the plate are expected to differ along their thickness following extended rule of mixture. Two different types of boundary condition are taken for plate analysis. First a static study of FGCNT plate is carried out and bending deflection are obtained for general boundary condition. The investigation is extended to bending of FGCNT plate integrated with piezoelectric layer. The effectiveness of piezoelectric materials on the regulation of nonlinear deflection of FG plate is presented. The influence of boundary conditions as well as CNT volume fraction on bending behaviour of smart plate is thoroughly examined.

2. Mathematical Model Formation

A smart FG plate integrated with piezoelectric layer on its top surface is shown (Figure 1a). Here ‘a’, ‘b’, and ‘h’ represent length, width and thickness respectively for FG plates. Top surface of FG plates is combined with piezoelectric material layer having length L_p and thickness h_p .

2.1. Modelling of FGCNT Reinforced Composite

The volume fraction is defined as follows (Figure 1) for the FGXCNT,

$$V_{CNT}(z) = \left\{ \begin{array}{ll} \frac{4z}{h} V_{tcnt} & \left(0 \leq z \leq \frac{h}{2} \right) \\ -\frac{4z}{h} V_{tcnt} & \left(-\frac{h}{2} \leq z \leq 0 \right) \end{array} \right\} \quad (1)$$

for the UDCNT,

$$V_{CNT}(z) = V_{tcnt} \quad \left(-\frac{h}{2} \leq z \leq \frac{h}{2} \right) \quad (2)$$

for the FGΔCNT,

$$V_{CNT}(z) = \left(\frac{1}{2} - \frac{z}{h} \right) V_{tcnt} \quad \left(-\frac{h}{2} \leq z \leq \frac{h}{2} \right) \quad (3)$$

Here the symbols X and Δ represent the material profiles [5].

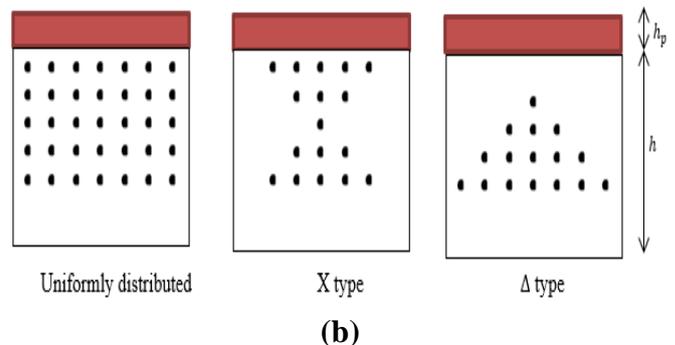
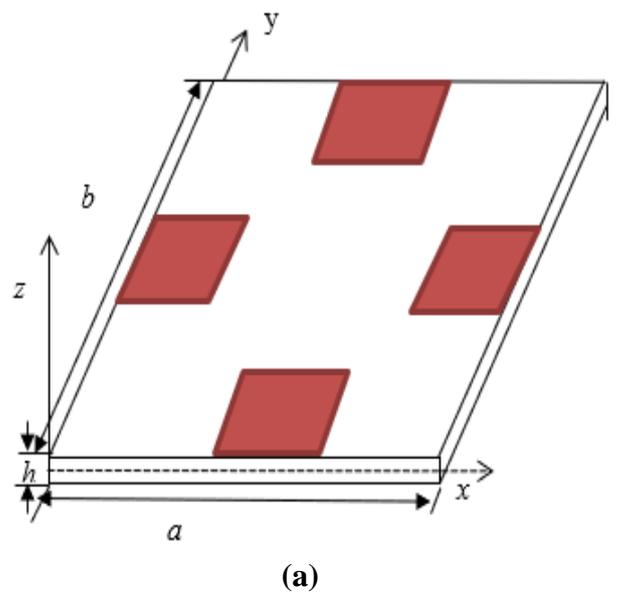


Figure 1 Schematic Representation of FGCNTRC Plate Integrated with a Layer of Piezoelectric Material

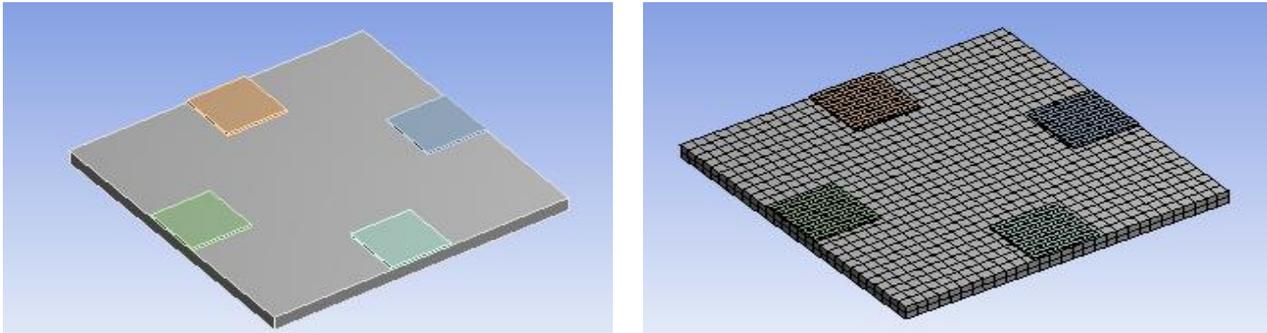


Figure 2 (A) Model and (B) Meshing of CNT Reinforced FG Composite Plates

Estimated volume fraction of CNT calculated with constituent's density can be expressed as [7]

$$V_{tcnt} = \frac{\alpha_{cnt}}{\alpha_{cnt} + (\rho^{cnt}/\rho^m) - (\rho^{cnt}/\rho^m)\alpha_{cnt}} \quad (4)$$

and estimated elastic properties of FGCNTRC layer are [17]

$$\begin{aligned} E_{11}(z) &= \eta_1 V_{CNT}(z) E_{11}^{cnt} + V_m(z) E^m \\ \frac{\eta_2}{E_{22}(z)} &= \frac{V_{CNT}(z)}{E_{22}^{cnt}} + \frac{V_m(z)}{E^m}, \\ \frac{\eta_3}{G_{12}(z)} &= \frac{V_{CNT}(z)}{G_{22}^{cnt}} + \frac{V_m(z)}{G^m}, \\ v_{12}(z) &= \eta_1 V_{CNT}(z) v_{12}^{cnt} + V_m(z) v^m \\ v_{21}(z) &= \frac{v_{12}(z)}{E_{11}(z)} E_{22}(z), \\ \rho(z) &= V_{CNT}(z) \rho^{cnt} + V_m(z) \rho^m, \\ V_m(z) &= 1 - V_{CNT}(z) \end{aligned} \quad (5)$$

where $()_{cnt}$ represents elastic properties of CNT fibers, $()_m$ indicates elastic properties of matrix materials. V_m and V_f are volume fraction of matrix and CNT reinforced fibres and η_1, η_2, η_3 are efficiency parameters of the CNT in Figure 2.

2.2. Development of Ansys MODEL

Functionally graded composite plate is developed taking various layers in ANSYS software and piezoelectric material layer is put over this FG substrate. The extended mixture rule is used to consider material properties of layers. Each layer is in perfectly matched with one other by applying tie constraint in the ANSYS software (Figure 2a). FG plate considered here for analysis is made up with carbon nanotube (CNT) reinforced with Polymethyl methacrylate matrix material. The model is discretize using SOLID 186 element as it exhibits

quadratic displacement behaviour and is also having spatial orientation. It also supports large strain capability, large deflection and stress stiffing. To achieve precise and accurate result, fine meshing is used. Fine meshing (Figure 2b) is applied for obtaining precise and accurate results. First, convergence analysis is done for optimized modeling purpose. A 10 layered configuration is considered optimally for developing the model. Discretization of the model is carried out by meshing the structure through manual size control and an optimal mesh size is selected based on the convergence analysis.

3. Results and Discussions

This section presents the results obtained by using the simulation model developed in ANSYS and obtained from experiments. Three types of square plates are considered with different boundary conditions and different width-to-thickness ratios (b/h) and their behavior under mechanical loading are investigated. The three types of CNTRC plates considered in the study are with uniformly distributed UD-CNTRC, FG- Δ type and FG-X type CNTRC. UD-CNTRC refers to unidirectional carbon nanotube reinforced composite, while FG- Δ , and FG-X CNTRC refer to functionally graded carbon nanotube reinforced composite plates with different configurations. The boundary conditions considered in the study include all edges simply supported (SSSS), all edges clamped (CCCC), two opposite edges simply supported with other two clamped (SCSC), and two opposite edges simply supported with other two free (SFSF). The behaviors of these plates under mechanical loading

for different b/h ratios are evaluated. The thickness of the plates is taken to be 2 mm and the width-to-thickness ratios considered in the study are 10, 20, and 50. The applied uniformly distributed load is $q_0 = -0.1 \text{ MPa}$. Results are computed and presented to study the impact of amount and distribution of CNT and the boundary conditions on the bending characteristics of FG CNT reinforced composite plate. Polymethyl methacrylate is taken as matrix material for a material point view whose properties are

$$E^m = 2.5 \text{ GPa}, \rho^m = 1190 \frac{\text{kg}}{\text{m}^3} \text{ and } V^m = 0.3.$$

The CNT properties considered are $E_{11}^{CNT} = 600 \text{ GPa}$, $E_{22}^{CNT} = 10 \text{ GPa}$, $G_{12}^{CNT} = 17.2 \text{ GPa}$, $\rho_{CNT} = 1400 \frac{\text{kg}}{\text{m}^3}$ and $V^{CNT} = 0.19$. The value of CNT efficiency factors used in this work is continuation of previous work is shown in Table 1. The mechanical properties of piezoelectric materials are

$$C_{11} = 132 \text{ GPa}, C_{55} = 26 \text{ GPa}, C_{12} = 71 \text{ GPa}, C_{13} = 73 \text{ GPa}, C_{33} = 115 \text{ GPa}$$

and electrical constant is

$$e_{33} = 14.1 \text{ Cm}^{-2}, e_{31} = -4.1 \text{ Cm}^{-2}, e_{15} = e_{24} = 10.5 \text{ Cm}^{-2}$$

and dielectric constant is

$$E_{11} = E_{22} = 7.124 \text{ nFm}^{-1}, E_{33} = 5.841 \text{ nFm}^{-1} \text{ and } \rho_{pizo} = 7500 \frac{\text{kg}}{\text{m}^3}.$$

Table 2 shows a comparison study of graded CNT reinforced composite plate with the existing results [7] which show good agreement.

Table 1 Efficiency parameter of CNT [7]

CNT efficiency parameter	V_{tcnt}	V_{tcnt}	V_{tcnt}
	0.12	0.17	0.28
η_1	0.137	0.142	0.141
η_2	1.022	1.626	1.585
η_3	0.715	1.138	1.109

The accuracy of the results obtained from their models is dependent on factors, such as the choice of elements, material properties, boundary conditions, and modeling assumptions. The accuracy of the results obtained from their models is dependent on factors, such as the choice of elements, material properties, boundary conditions, and modeling assumptions.

In Table 3, the findings for clamped smart FG plates with different material profiles are shown. As can be seen, with and without the supply of voltage, the centre deflections of plates with X type grading are at their lowest at $V_{tcnt}=0.28$ and $L/h=10$. Additionally, it can be seen that a higher CNT content in the composite, up to a specific level, enables it to handle heavier loads with minimal deflection.

Table 2 Maximum Deflections Using Present Simulation Model and Available Results [7] for UD Type Material Profile

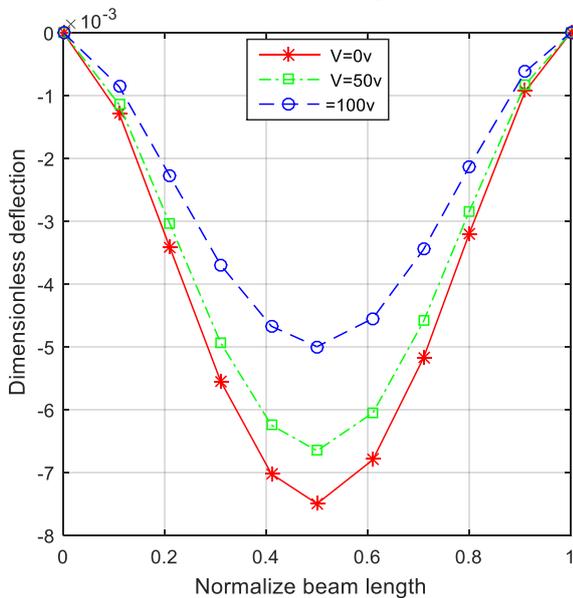
V_{tcnt}	0.11			0.14			0.17			
	a/h	10	20	50	10	20	50	10	20	50
Experi-mental		3.679×10^{-3}	3.578×10^{-3}	1.139	2.758×10^{-3}	2.897×10^{-3}	0.9095	2.182×10^{-3}	2.274×10^{-3}	0.7412
ANSYS		3.709×10^{-3}	3.603×10^{-3}	1.147	2.7965×10^{-3}	2.949×10^{-3}	0.9135	2.288×10^{-3}	2.311×10^{-3}	0.74635
Ref. [7]		3.739×10^{-3}	3.628×10^{-3}	1.155	3.306×10^{-3}	3.001×10^{-3}	0.9175	2.394×10^{-3}	2.348×10^{-3}	0.7515

Table 3 Maximum Deflections of Smart FGCNTRC Plate Integrated with Piezoelectric Material Layer (CCCC)

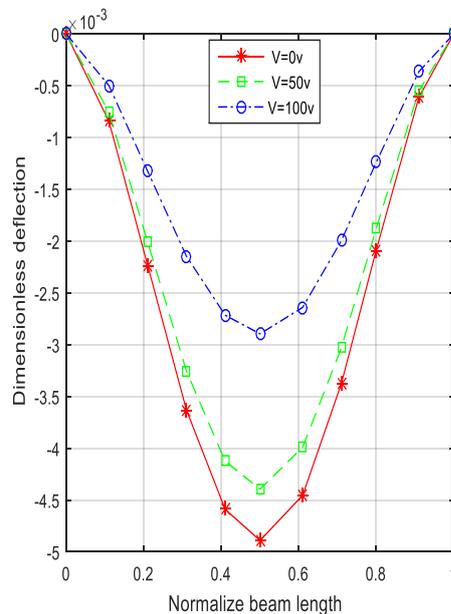
L/h ratio	Material profile	$V_{tcnt}=0.11$		$V_{tcnt}=0.17$		$V_{tcnt}=0.28$	
		V=0v	V=100v	V=0v	V=100v	V=0v	V=100v
10	uniform	3.645×10^{-3}	3.221×10^{-3}	2.314×10^{-3}	2.153×10^{-3}	1.974×10^{-3}	1.632×10^{-3}
	Δ type	4.354×10^{-3}	3.754×10^{-3}	2.823×10^{-3}	2.594×10^{-3}	2.543×10^{-3}	2.194×10^{-3}
	X type	3.105×10^{-3}	2.652×10^{-3}	1.916×10^{-3}	1.631×10^{-3}	1.514×10^{-3}	1.191×10^{-3}
20	uniform	3.547×10^{-2}	3.023×10^{-2}	2.225×10^{-2}	1.956×10^{-2}	2.012×10^{-2}	1.550×10^{-2}
	Δ type	4.847×10^{-2}	4.153×10^{-2}	3.118×10^{-2}	2.645×10^{-2}	2.741×10^{-2}	2.215×10^{-2}
	X type	2.6451×10^{-2}	2.172×10^{-2}	1.527×10^{-2}	1.275×10^{-2}	1.224×10^{-2}	0.814×10^{-2}
50	uniform	1.1231	0.8314	0.7411	0.4327	0.5137	0.2121
	Δ type	1.9545	1.3481	0.9918	0.6123	0.7431	0.4132
	X type	0.7421	0.4121	0.5001	0.3172	0.4004	.0953

It has been found that CNT increases strength, which lowers transverse deflection, with increasing CNT content. Additionally, the results shown in Tables 3 show that the piezoelectric material layer can reduce plate deflection when supplied with power (V=100 volts). When compared, it is also

seen that the Δ type plate exhibits the highest deflection while the X type graded plate exhibits the lowest deflection. This is because the X type graded plate exhibits more stiffness than the Δ type due to its symmetrical profile in Figure 3.



(a)



(b)

Figure 3 Centerline Deflection of Smart Composite Plate Subjected to Different Input Voltage (A) CCCC Boundary Condition (B) SSSS Boundary Condition

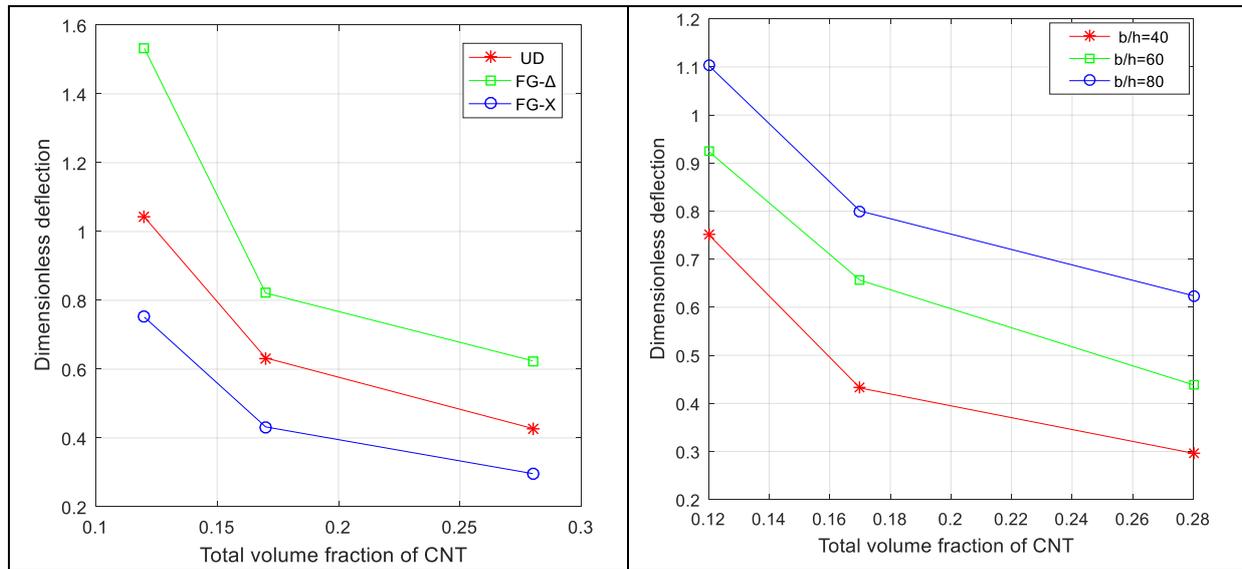


Figure 4 (A) Dimensionless Central Deflection of FGCNTRC Plate for (A) Different Distribution (B) Different Volume Fraction

The dimensionless deflection of the FGCNTRC beam is shown in Figure 4 (a) for various CNT volume fractions, and it can be shown that as overall CNT fraction increases, dimensionless deflection for the active patches ($V \neq 0$) decreases. The dimensionless deflection of the FGCNTRC beam for various aspect ratios is shown in Figure 4(b), which also illustrates that an increase in deflection is seen for an increase in aspect ratio for the active patches ($V \neq 0$).

Conclusions

Bending analysis of plate of functionally graded CNT reinforced composites with piezoelectric materials is performed. The CNTRC layer material properties were determined using extended mixture rule considering three different profiles of distribution. The micromechanics principle is expanded to quantify the effective properties of material of the FGCNT composites by creating a simulation model in commercially available ANSYS software. After verifying the reduced model with the help of previously published results, bending analysis of FGCNTRC plates integrated with piezoelectric materials is performed. It has been noted that

- For increased CNT volume fraction, the overall deformations are decreased for the smart

FGCNTC plate. The smart plate shows a reduced deformation for uniform loads and for all possible boundary conditions with the activated patch of piezoelectric material.

- The dimensionless maximum deflections of Smart FGCNTRC plate of X type profile distribution is minimum for $V_{\text{cnt}}=0.28$ and aspect ratio of 20 for both activated as well as inactivated piezoelectric patches.

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