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Traces of Augmented Reality in Learning Mathematics: A Survey

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Abstract
Augmented reality(AR) has been widely explored in the fields of gaming, medicine, advertising and social media. It is gradually penetrating into the field of education. The paper tries to address the gap in the methodology and concepts utilized so far and reviews the recent trends of AR in mathematical education. AR based learning methods are available in teaching various subjects such as physics, chemistry, although not very popularly implemented. AR approaches are available in mathematical teaching as well but majority for school curriculum. This study refers to the various zones of analysis to get an aerial overview of the quantitative study. The key factors for implementation and adaptation of the technology are backed by factors such as high cost, technology glitches, health issues and challenges to bring the correct collaboration of technology with the classical teaching methodologies in classrooms. Here a crisp list of tools utilized for mathematical education purposes is studied. The keyword search narrowed approach is tabulated with article counts. It aims to make mathematics easily accessible and understood by a larger community of remotely located students. In view of exploring and understanding the penitration and furthur scope of infiltration of augmented reality in education of mathematics, the paper gives an overall review of the concept and its recent statue of advancement.

Keywords: *Augmented Reality, AR Applications, Computers and Education, Mathematics, Research.*

1. Introduction
According to Cipresso et. Al. (2011) [1], Augmented reality (AR) is an interactive experience that combines the real world and computer-generated 3D content. The content can span multiple sensory modalities, including visual, auditory, haptic, somatosensory and olfactory. Wu, Hsin-Kai et. al. [2] (2013) states that AR can be defined as a system that incorporates three basic features: a combination of real and virtual worlds, real-time interaction, and accurate 3D registration of virtual and real objects. Thus, AR would require a device to offer a real world environment in a physical sense. The sound effects, videos and images provide a real feel to the user. It could be a direct view or an indirect view. Virtual reality (VR) is an immersive experience which is done using a headset and it provides a digital twin to real world experiences through sound, images and other features. AR you experience both real and virtual simultaneously whereas VR gives the user a totally immersed experience. An extended reality refers to combining both and is achieved by interactions between computer technology and wearables. Instagram is a medium which shares AR effects. Lenslist is like an archive, or rather an encyclopedia of Augmented Reality filters. They regularly spotlight the best AR Effects which helps the developers to improve the platform. Due to this virtual interactive experience AR has been utilized in many areas of education in various subjects such as chemistry, biology, statistics and mathematics. AR is nowadays used in various fields and its applications come in variety. AR applications range from Education to Medicine to Entertainment to Gaming to name any field. The paper deals with identifying various genral AR applications and narrowing down to detailed analysis of mathematical applications. It traces the path of research done in recent years and highlights the advances to be included in future work.

2. Motivation
The literature found with patent is as old as 1997, Devices such as a remote learning system combine

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on-line service information and remote user information with a television signal designed to be received by students using multimedia personal computers, developed by Guy Frederick and Roberts Ronald [3]. The combined signal is then transmitted to all of the students at their personal computers. At each personal computer, the received signal is separated into its component parts by a VBI modem for display or control on/off the personal computers. Each personal computer is capable of communicating back to a main server over a network such as a public telephone network. Suzhou Ed J [4] made a situation simulation support system towards mathematical education, including situation emulation support module, problem situation emulation module, and solution approach error correction and reminding module, extension knowledge tracing model analysis module. Wernhuar Tarng, Yu-Cheng Tseng, & Kuo-Liang Ou [5] created an augmented reality system to assist high school students in learning chemistry. Students could use AR cards to conduct virtual chemistry experiments, and the submicroscopic view of a chemical reaction were displayed according to the chemical equation specified by the reactants and coefficients on AR cards.AR system was more effective with students as compared to the traditional approach. G Pavlendova[6] developed a physics based ARphymedes plus for special children. It's a story-based learning using AR connected to physics learning.AR is a combination of experiments, visualization and calculation process and it gives implemented results to the user as described in Figure 1. The AR technology enables the contents to overlay to the real environment and the user is capable of interacting and experimenting with the virtual environment. Researchers have studied the applications of AR in mathematics as shown in Figure 2.

Figure 1 Overview of Inputs and Output in AR Implementation

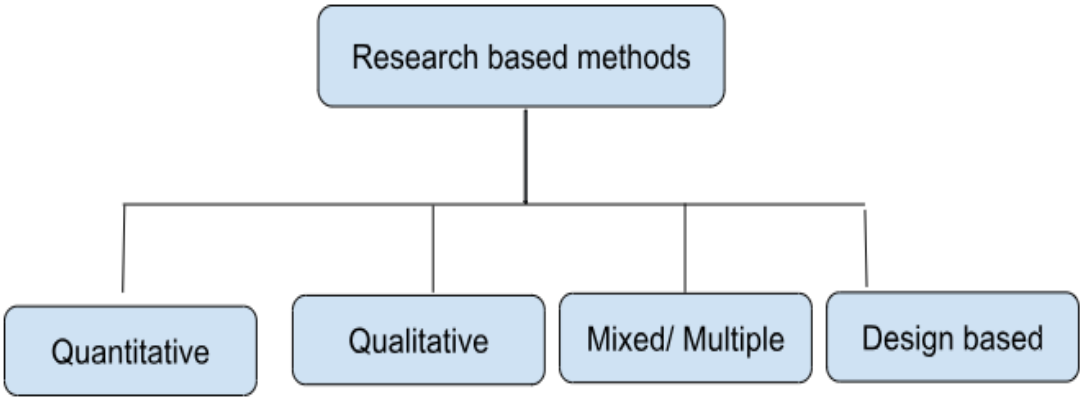

Figure 2 Research Based Methods and Classification

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Several researchers have performed qualitative, quantitative, mixed and design-based study on tools of AR for educational purposes.Greefrath G, Hertleif C, Siller H S [7] discusses to what extent modeling processes using digital tools can be described theoretically, and surveys significant empirical findings in this field. Based on a quantitative control study with 709 students. Vanessa Ivan, Siti Mistima Maat [8] studied the impact of the use of AR in mathematics education from the perspective of students' cognitive and affective development. Timur Koparan, Hakan Dinar, Ezgi Taylan Koparan, Zeliha Sema Haldan [9] designed, developed and revealed the effect of an AR material to improve spatial ability in secondary school students using mobile devices. Waterfall model was implemented for the study.Mustafa Cevikbas [10] has reviewed and described in detail the methodologies and digital tools used in the studies which include smartphones, glass headphone collectors, 3D printers, projectors, MP3 players, checklist, guidelines booklets, magic box, marker based systems and so on. The softwares included unity, Vuforia, HP reveal, GeoGebra, Zapper, Neotrie, Krpano, Blender, PhET, VisualMath and so on. Pratiti Sarkar [11] studied the content, context and design and focused on the handheld ARLE for classrooms.

3. Research Questions to Be Addressed

The study tries to give a brief on the recent implementations of AR particularly on mathematics learning, it ponders on the questions such as

- Which software's are utilized for AR implementation?
- Who are the beneficiaries for the tools developed and the experimental sizes the experiment is conducted?
- How much of research is done in various other subjects as compared to mathematical implementation?
  The devices developed and availability for which course content?

4. Data Collection

4.1 AR and its related applications

In our study, we searched for research papers containing the keyword "Augmented Reality Applications" with different possible sectors. Performing serach using random keywords gave a huge pool of research papers. On narrowing down the search using specific word arrangement and using proper conjunction "AND" we could manage to get a good appropriate result. The number of articles found were taken by further applying advanced filter on the year as "From 2023 To 2024". The results of the search query is noted in the table below. has reviewed and described in detail the methodologies and digital tools used in the studies which include.Figure 3 describes the graph of the keyword based search made using various search phrases

Table 1 Table showing the number of articles found in Google Scholar and Google Patent For eg: "Augmented Reality" AND "Applications"

Keywords searched	Google Scholar	Google Patent
Applications	21,300	22,521
Mobile	5160	16,609
Virtual	6,230	20,214
Education	5,010	746
Manufacturing	2,550	6,022
Medicine	1,960	293
Location-based	884	1,765
Design	6,160	11.681
Tourism	1,550	97
Healthcare	1,970	999
Entertainment	1,840	4,418

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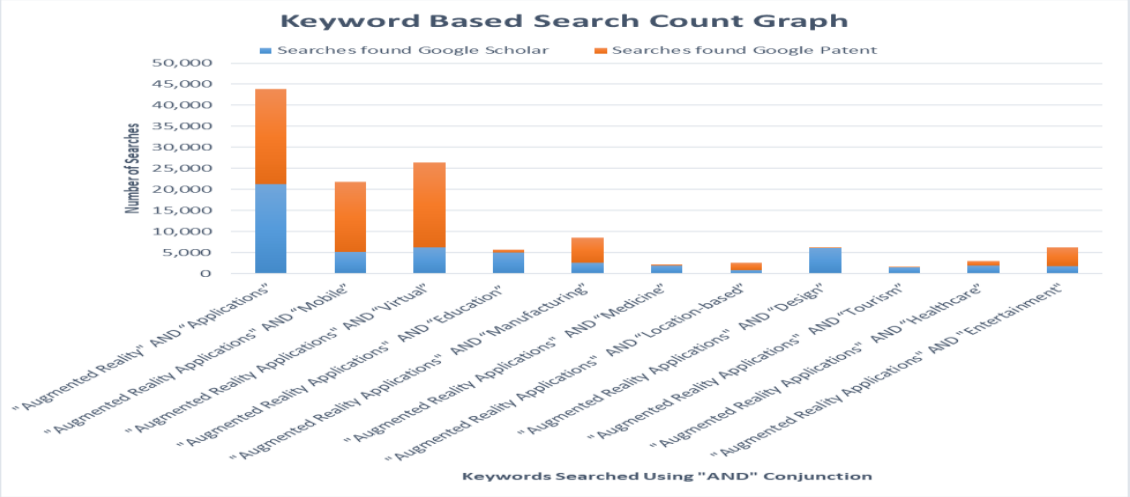

Figure 3 Keyword Based Search Showing the Usage of "AND" Conjunction

4.2 AR Applications in Educational Domains

Narrowing the keywords from AR Applications in various fields to AR applications in Education, we found a good research opportunity for development in the education field. The below table gives the results of the search carried out for AR in Education. Education itself is vast in itself, various subject areas were considered for search to find out how much work has been done in different subjects that are taught at school level. The briefing of the search carried out is noted in the table 2 given below. From the above table it can be inferred that within a span of two years of research in the AR field many educational areas have been covered.

Table 2 AR applications in Education Field: Augmented Reality is searched with other domains. For eg. "Augmented Reality" and "Mathematical Education"

Keywords Searched	Google Scholar	Google Patent
Mathematical Education	580	7
English Education	2,320	35
Science Education	9,260	50
History Education	547	52
Geography Education	297	4
Media Education	734	16
Arts Education	726	14
Biology Education	1,030	2
Chemistry Education	1,120	2
Physics Education	1,670	0

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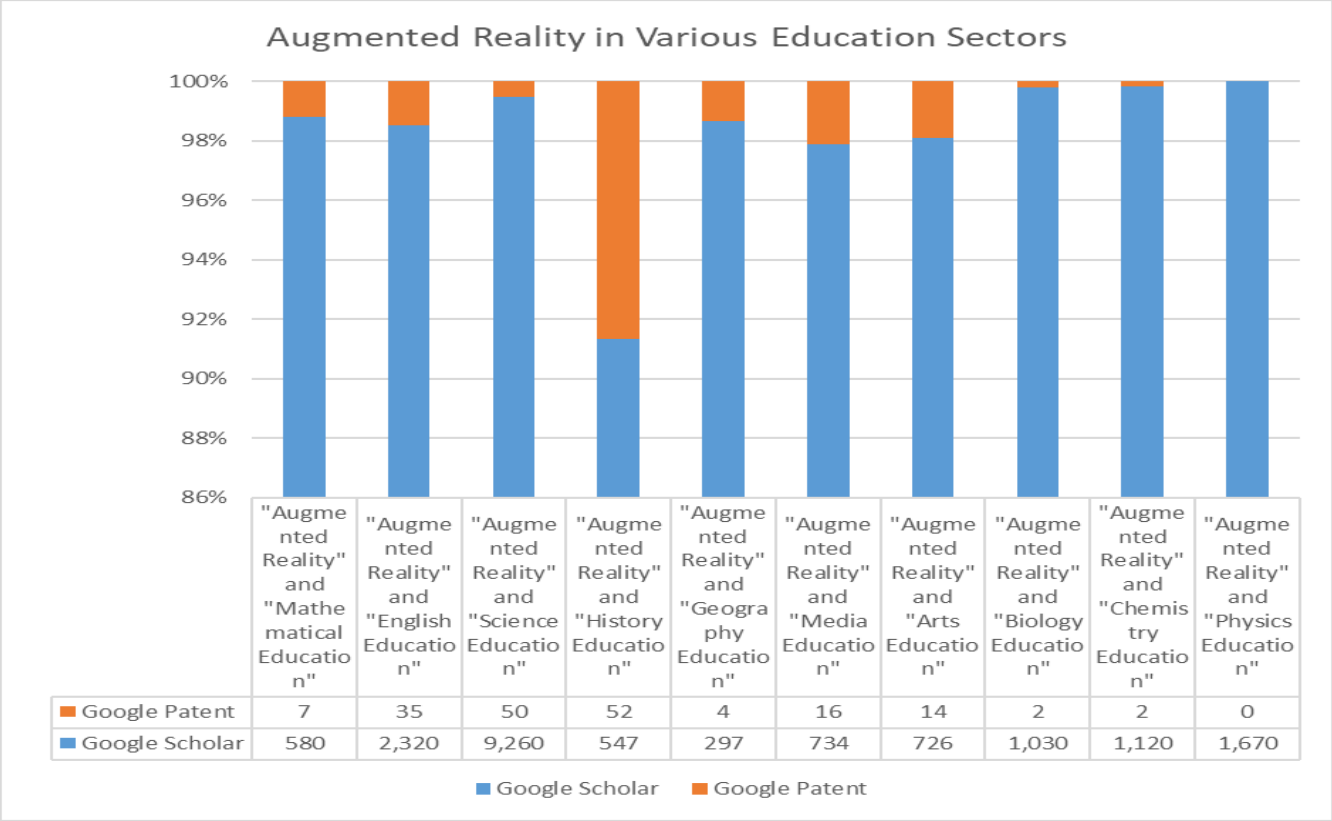

Figure 4 Graphs Showing Research Carried Out in Various Educational Fields

4.3 Advances in App Development for Math Learning

With advances in technology, many tools, apps, web apps have been developed keeping Math in mind. As per our study, more than 35+ tools, web apps, mobile apps are available that help in better learning for the students and teaching mechanism for the teachers. Some of them are freely available for use whereas some are available on chargeable basis. Out of the many tools available, the table below records a few widely used Math tools not only by students for learning purposes but also by teachers for teaching purposes [12-26].

Table 3 Mathematical Tool/Web App/ Mobile App Mentioning Its Usage

Tool Name	Usage
GeoGebra [12]	Dynamic Mathematical software program that covers various mathematical topics such as Geometry, Algebra, Spreadsheets, Graphs, Calculus and many more. The community is progressing and are top providers of dynamic mathematical software
Geometry Pad [13]	An app developed to learn geometry and its various constructions in a fun based manner. Specifically designed for iPads and Tablets
Photomath [14]	It's an app that captures the image of a math problem and provides the step by step solution to it. The steps are explained just like a teacher explains the steps in a math problem
Khan Academy	This app uses instructional math videos, practice exercises, and has a personalized learning dashboard for individual students so that they can study at their own pace. Apart from Math, this

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[15]	app is also available for other subjects such as Science History, etc.
Shapes 3D [16]	Shapes 3D is an AR app that teaches geometry to the students by picking various figures for building simple to complex geometrical figures.
CK 12 [17]	This app offers adaptive practice, workbooks, quizzes, tests and several simulations.on various math related topics
Buzzmath [18]	A web-based application that motivates the students to remain focused by providing more than 3000+ practice problems till the learner is thorough with their Math concepts.
Cuethink [19]	An app developed for students with the age group of 2 - 12 supports weak students to learn better and keep motivated students who are already doing well
FluidMath [20]	Fluidmath is a kind of smart paper where teachers can create dynamic instructional material for the students. The tool is created for use on Pen-Centric platforms such as tablets and whiteboards.
Rocket Math [21]	This app comes in two flavors; one printable sheets and other in the form of a game. The printable sheets are given to students to work in pairs whereas the game comes with a level to cross. As the student progresses in the game, the difficulty level of the problem also increases. Here teacher intervention is not required.
Desmos [22]	This tool is a great learning tool and offers the following facilities: • Used as a graphing calculator • Accessible to Visually impaired students also • Small math games are available • Students can create their own math ideas • Teachers can create material for the students through the activity builder feature.
Mathalicious [23]	Using this app, Math is taught using application orientation. This app enables the development of critical thinking skills of the students by solving real world application problems.
Prodigy Math Game [24]	It is a Pokémon based online math game, created for students of grades 1 to 8 where students have to create their own avatar and learn as well as practice through games.
Mathspace [25]	The Mathspace app is designed for students ranging from age 7 to 18. Math problems are given and as the students solve them, the difficulty level of the problems increase making the students work harder for solving the problem. The app maintains a history of the student performance and on the basis of this history problems and its difficulty level is suggested.
Brainly [26]	Brainly is a community- based app solving for the students' problems on various subjects. The students post their queries and people of the community respond to the questions asked by the students. It is more like a forum used for solving various problems.

4.4 AR and Its Effects in Mathematics Learning

A systematic review was carried out by FA Hidajat et. al. (2023) [27] using the principles of National Council of Teachers of Mathematics (NCTM). After applying various filters, they analyzed 66 papers. The NCTM principles, guidelines and expectations analyzes Mathematical education into six themes such as equity, mathematics curriculum, mathematics teaching, learning, assessment, and mathematics technology. From the survey carried out by [a] it was observed that AR has a lot of potential in teaching mathematics in a creative manner. Also, the features

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of AR that are prepared using tools such as Unity and Vuforia are quite promising and have a positive impact on the quality of mathematical education. Yao et. al. (2024) [28] studied the effect of AR supported learning methods for Math and English Language. The study was carried out for 26 pre-school children. Marker-based AI techniques were used to enrich the children's learning experience. The group of children were divided into control groups (learning through traditional methods) and experimental groups (learning through AR-based methods) and observations were made. Results demonstrated not only children belonging to the experimental group enjoyed learning through AR, but teachers also found the AR designs to be attractive to make children learn.GeoGebra(a free Math Tool) [12] when integrated with AR provided a good learning experience and better performance to school students. This experimental method was carried out by IparraguirreVillanueva, Orlando, et al. [29], where they studied 78 students and found that 70% of the experimental group had outstanding performance while the remaining 30% students could reach the expected level of learning. Considering the controlled group that were taught without AR integration 10% achieved expected level performance, 85% were progressing and 5% were still in their initial stage. In summary, the researchers found the integration of AR with GeoGebra improvised the learning of the students.Hwang, Wu-Yuin, et al [30] developed an app titled Authentic GeometryGo abbreviated (AGG), to facilitate the students to learn geometry concepts. The study included experimental and control group students and were taught using AGG app and traditional method of pen and paper respectively. The experimental group students outperformed the control group. The study concludes with note to use AR features with different learning activities to enhance the learning of the students.An augmented reality-based application, 'Hyperspace', was developed by Singh, Gurwinder, et al. [31] to enhance the spatial skills and conceptual knowledge in trigonometry of first year engineering students. 3-D trigonometric functions were dynamically generated using procedural content generation algorithms and computer graphics in AR. 127 students were randomly divided into control groups and experimental group. On evaluation it was observed that experimental group students outperformed the control group students, showing that "Hyperspace" application helped students to

Table 4 Specifications for The Papers

Authors	Area of Application	Reserach Methodology	Journal / Conference Proceedings	Beneficiary
FA Hidajat et. al.	Creativity in Mathematics teaching learning process	Literature Review	Journal of Computers in Education, 2023 - Springer	Research Scholars
Yao, Wei, Lei Wang, and Deyang Liu.	Math and English learning	Experiential Based	Universal Access in the Information Society (2024) - Springer	26 Pre-School children
IparraguirreVillanueva, Orlando, et al	GeoGebra (Math tool) integrated with AR for teaching Geometry	Experiential Based	International Journal of Engineering Pedagogy 14.3 (2024)	78 School children
Singh, Gurwinder, et al.	"Hyperspace" Application to teach rigonometric Functions	Experiential Based	Multimedia Tools and Applications (2023): 1-22.	127 First year Undergraduate Students
Hwang, Wu-Yuin, et al.	Authentic GeometryGo (AGG) app to help learn and assess geometry with the help of AR	Experiential Based	IEEE Transactions on Learning Technologies (2023).	Students learning Geometry

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Mustafa Cevikbas, Neslihan Bulut, & Gabriele Kaiser	Exploring the Benefits and Drawbacks of AR and VR Technologies for Learners of Mathematics	Methodology based	Recent Developments. MDPI	School children
Pratiti Sarkar, & Jayesh S Pillai.	Approaches for Designing Handheld AR Learning Experiences for Math Classrooms	Experimental based	Proceedings of the ACM on Human-Computer Interaction	32 participants from school
Suzhou Ed J.	A kind of situation simulation support system towards mathematical education.	Experimental based Patent	Patent	
Greefrath G, Hertleif C, Siller H-S	Mathematical modeling with digital tools a quantitative study on mathematising with dynamic geometry software ZDM Mathematics Education	Experimental based Patent	Patent	
Vanessa Ivan, Siti Mistima Maat	The Usage of Augmented Reality Technology in Mathematics Education	Experimental	A Systematic Literature Review International journal of academic research in progressive education and development 2024	School children

5. Articles Reviewed for Analysis

The below Table 4 indicates the analysis done on the recent research specifing the area of applications, research methodology, beneficiaries to understand the utility of AR in teaching mathematics to observe the advantages and gaps in applications done so far.

Conclusion

This article gives a glance of the recent advances in mathematical teaching and learning trends. Tools developed are functional in isolation and their complete potential is still unutilized. The study shows that patents granted in AR utility in history education are the highest as compared to mathematics, science, English, geography media and arts. The reason for this could be the demand and understanding of the history is far more adaptive to the technology as compared to the formative scripts.Research articles in AR for science education outnumber the AR applications in mathematics, science, geography, and History and Arts education. New advances which may lead to inclusion of emotional and language processing systems to guide the students with step-by-step hints. The authenticity of the solution pattern is highly recommended. Corpus expansion would lead to further improvement and inclusion of a wide syllabus that may cater to the needs of indigenous students' communities is the need of the hour. Collaborative and community research would benefit many students as a whole. The study paves the way for future research for the budding inventors with a specific subject specialization in mathematics.

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[15]. "Khan Academy" https://www.khanacademy.org/

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