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Numerical solution of Rössler attractor using power series method

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Abstract

Rössler attractor, topologically most simple chaotic attractor, involves three simultaneous equations, out of which one is non-linear. It has its behaviour largely dependent on the three constant parameters - a, b and c. To obtain its numerical solution a twofold approach was adopted. Its solution by power series was matched with that obtained by x-cos block methodology. To generate the three-dimensional output Scilab simulations freeware was practiced. Constants' determination was made for the set(s) of a, b and c, where the aperiodic nature was threshold.

Keywords*: Rossler Attractor, Differential equations, Power series, Scilab.*

1. Introduction

Chaotic dynamical systems refer to a class of mathematical systems that exhibit sensitive dependence on initial conditions and display complex, unpredictable behaviour over time. The behaviour of chaotic systems is deterministic, that means their future states are entirely determined by their current state and the governing equations of the system. However, due to their sensitivity to initial conditions, even a slight change in the starting state can lead to significantly divergent trajectories. Chaotic systems can be found in various fields, including physics, biology, economics, and computer science [1-3]. Some well-known examples of chaotic systems include the Lorenz system, the double pendulum, and the logistic map. The study of chaotic dynamical systems has important implications across disciplines. Chaos theory provides insights into the inherent limits of predictability, the emergence of complex behaviour from simple rules, and the sensitive interplay between determinism and randomness. It has applications in weather forecasting, population dynamics, cryptography, signal processing, and many other fields. Overall, chaotic dynamical systems represent a fascinating and intricate area of study that continues to capture the interest of researchers and scientists due to their rich mathematical structure and real-world implications.Power series solutions are a method of solving differential equations or finding approximate solutions to functions using a series expansion. A power series is an infinite series of the form:

$$f(x) = b_0 + b_1 (x - x_0) + b_2 (x - x_0)^2 + b_3 (x - x_0)^3 +.$$

where b_0, b_1, b_2, ... are coefficients and x_0 is a centre point. The power series is typically centred around a point where the function is well-behaved, often chosen as $x_0 = 0$ for simplicity.To use power series to solve a differential equation or find an approximate solution, we substitute the power series into the equation and equate the coefficients of each power of x. This process generates a recurrence relation between the coefficients, which can be used to determine their values [4].1The Lorenz and Rössler attractors are two famous examples of chaotic dynamical systems that were discovered independently in the 1960s. These systems are characterized by nonlinear differential equations and exhibit complex and unpredictable behaviour over time.2The Rössler attractor was discovered by Otto Rössler as a simplified model to study chaotic systems. It consists of three nonlinear differential equations

$$\frac{dx}{dt} = -y - z$$

$$\frac{dy}{dt} = x + ay$$

$$\frac{dz}{dt} = b + z(x - c)$$

Similar to the Lorenz attractor, x, y, and z represent the state variables, while a, b, and care system parameters. The Rössler attractor is characterized by a twisted, ribbon-like structure in three-dimensional space. Its trajectories are chaotic and exhibit sensitive dependence on initial conditions. The Rössler attractor is typically visualized in three-dimensional space, with x, y, and z as the axes.The Rössler attractor has found applications in various fields, including physics, biology, and engineering. It serves as a useful model for studying and understanding chaotic behaviour in dynamical systems [5]. The attractor's unique properties make it an interesting subject for research and analysis in the field of chaos theory. Numerical solution of Rössler attractor has been already obtained by Xcos method. In Current study Rössler attractor was solved using the power series method and its three-dimensional output is being discussed.

2. Materials, Methods and Apparatus

2.1 Numerical methods

We are using Scilab, it is a freeware and open-source software for numerical computation and simulation of various mathematical, physics, engineering related problems.Scilab is a high-level programming language which has the capability to solve many problems like 2D, 3D simulations, Dynamic System modelling (Xcos), Control Systems, Signal Processing Problems etc [6-7].We going to use two ways to understand the problem in hand, one is that we are going to write a program in Scilab for power series solution of Rossler Attractor and simulate it to get an 3D output and second, we are going to use Xcos to make a Dynamic System for Rossler Attractor to check the validity of the program written.Rossler Attractor is a chaotic system which is represented by three nonlinear ordinary differential equations given by:

$$\frac{dx}{dt} = -y - z$$

$$\frac{dy}{dt} = x + ay$$

$$\frac{dz}{dt} = b + z(x - c)$$

Where a, b, c > 0, are parameters.

These equations represent continuous time dynamic system.

Rossler attractor has been solved by many different numerical methods like Runge-Kutta method etc.

We discuss the method of solving Rossler Attractor using power series.

We define:

$$[x] = \sum_{i=0}^{N} A_i \tau^i ;$$

$$[y] = \sum_{i=0}^{N} B_i \tau^i ;$$

$$[z] = \sum_{i=0}^{N} C_i \tau^i$$

Now Substituting these power series in Rossler Equation we get, a recurrence system:

$$A_{i+1} = \frac{-TB_i - TC_i}{\phi_{1i}}$$

$$B_{i+1} = \frac{TA_i + a^* B_i}{\phi_{1i}}$$

$$C_{i+1} = \frac{b^* + TS_i - c^* C_i}{\phi_{1i}}$$

Where a^*=a T, b^*=b T and c^*=c T and the Cauchy product is given by

$$S_i = \sum_{p=0}^{i} C_P A_{i-p}$$

Also, τ=t/T, ϕ_1i is the index till which iteration is needed.

Now, using above formulation we arrived at an iterative solution using power series, now simply

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using initial conditions on a, b, c and x0, y0, z0 which are arbitrary [8]. Numerical output was obtained using a = b = 0.1 and c is varied to see various output, while x0 = 6, y0 = -3 and z0 = 0. Outputs are very sensitive to initial conditions as a result it possess a challenge to any numerical tool.Now, we use available blocks from palette to design a dynamic system in Xcos in Scilab using conventional way of solving Rossler attractor using available numerical method like Newton, Runge-Kutta etc. to check the power series formulation.

3. Result and discussions

Two outputs are considered for comparing. One generated using program in Scilab and another by devising a dynamic system in Xcos [9]. This will basically be used to understand the result using power series method to already available solutions.

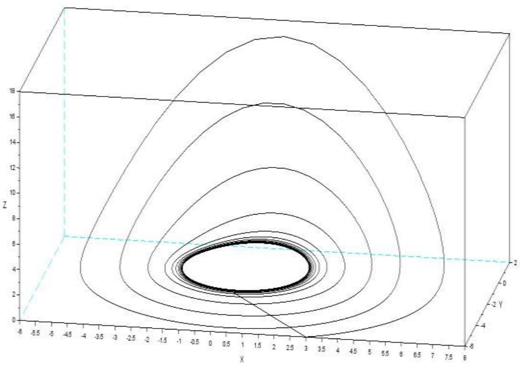

Figure 1 a=b=0.1, c=1(Power Series)

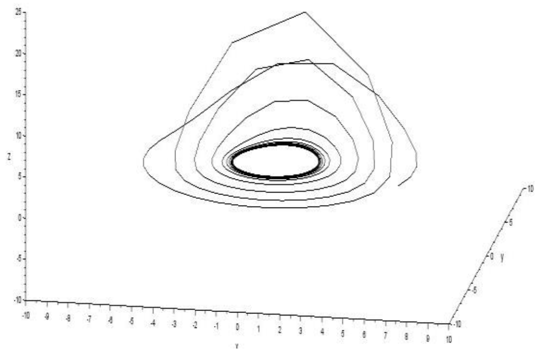

Figure 2 a=b=0.1, c=1(Xcos)

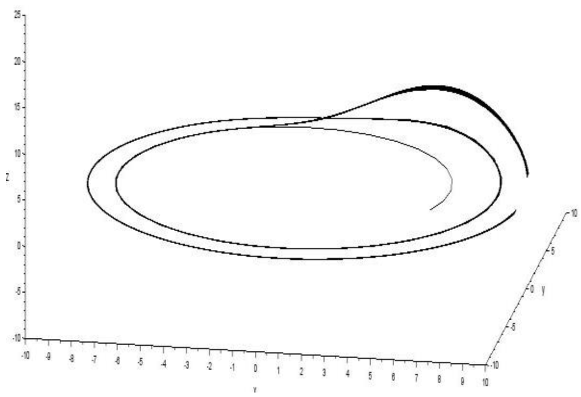

Figure 3 a=b=0.1, c=6(Power Series)

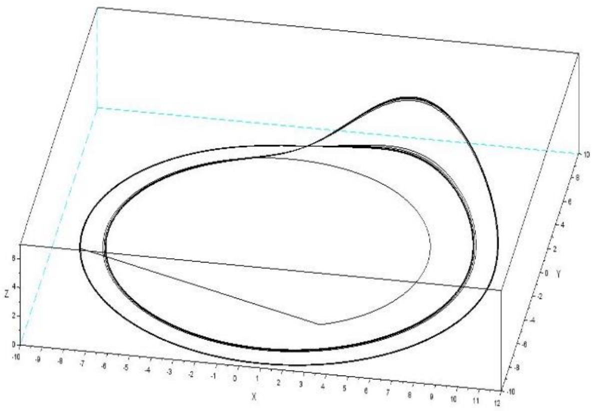

Figure 4 a=b=0.1, c=6(Xcos)

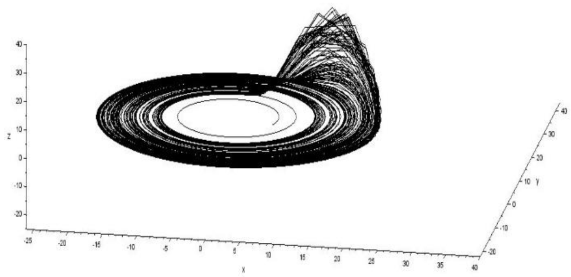

Figure 5 a=b=0.1, c=14(Power Series)

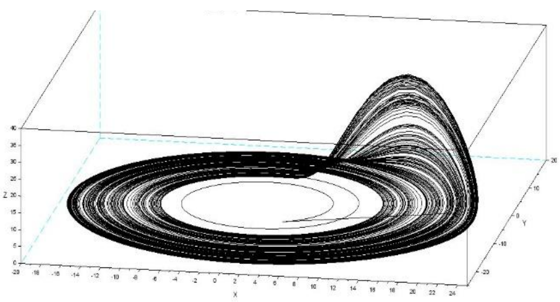

Figure 6 a=b=0.1, c=14(Xcos)

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Figure 1, Figure 3 and Figure 5 represents the results by applying power series solution, while Figure 2, Figure 4 and Figure 6 represents already available solutions using Xcos methodology. In our results we can see that curves are highly coherent and also Xcos result approaches the power series solution to Rössler attractor for a = b = 0.1 and c = 6. [10] It is observed that for same number of iterations power series solution gives higher approximation to required problems.In recent studies these are applied to Model Radio Frequency Interferences, Image encryption, Studying Heart Sound and Synchronization, Secure Digital Communication Systems, Generation of Cryptographic keys etc.3-6.For these processes small amount change in data can change entire system of study, for this reason we suggest using power series solution for superior understanding given the limits of existing methods[11].

Conclusion

Rössler attractor, was solved using the power series method in Scilab environment. The output generated are visualized and then compared with that obtained by Xcos methodology. Comparison shows good coherency and using Power series solution to Rössler attractor can give better approximation for required iterations. Constants' determination was made for the set(s) of a, b and c, where the aperiodic nature was threshold and found to be c = 4 and a=b=0.1 within the computational limits[12-15].

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