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Exploring the Role of Queueing Theory in Manufacturing: An Analytical Study

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Abstract
Manufacturing system is a combination of humans, machinery and equipment that are bound by a common material and information flow. Queueing theory plays a pivotal role in optimizing manufacturing processes by analysing and modelling the flow of entities, such as materials or tasks, within production systems. This article delves into the significance of queueing theory in manufacturing, highlighting its applications in enhancing efficiency, reducing wait times, and minimizing resource wastage. By examining factors like arrival rates, service times, and system capacities, queueing theory enables the identification of bottlenecks and the design of strategies for smoother operations. This work provides a valuable insights of queueing theory, guidance for decision-making, streamline workflows, and ultimately improves the overall manufacturing performance.
***Keywords**: Queueing theory, Manufacturing processes, Production systems, Waiting lines, Resource allocation.*

1. Introduction
Queueing theory, a branch of operations research and applied mathematics, holds a crucial position in the realm of manufacturing optimization. In manufacturing systems, the flow of entities such as products, materials, and tasks, is subject to varying degrees of uncertainty, leading to inefficiencies, delays, and resource underutilization. Queueing theory offers a systematic framework for analysing and understanding these dynamics, ultimately aiding in the enhancement of production processes. Abou Rizk et al. [1] have researched in modelling and simulation holds significant potential for advancing the field of construction engineering operations. By leveraging sophisticated computational tools and techniques, researchers and practitioners can gain valuable insights into the complex dynamics of construction projects, leading to improved efficiency, cost-effectiveness, and overall project outcomes. Through the application of modelling and simulation, various aspects of construction engineering operations can be analysed and optimized. These include project scheduling, resource allocation, risk assessment, and decision-making. By creating virtual representations of construction projects, researchers can experiment with different scenarios, test hypotheses, and evaluate the potential impact of various strategies before implementation. One key advantage of modelling and simulation is its ability to handle the intricacies of large-scale projects with numerous variables and interdependencies. Researchers can simulate real-world conditions, taking into account factors such as weather, resource availability, and unforeseen delays. Manufacturing environments are characterized by intricate interactions among different stages of production, equipment

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availability, workforce allocation, and task prioritization. These complexities often result in queuing phenomena, where entities wait in line before they can be processed or moved to the subsequent stage. Understanding and managing these queues are essential to minimize production lead times, improve resource allocation, and optimize overall system performance. Armero et al. [2] have proposed Bayesian hierarchical models in manufacturing bulk service queues. These models help to reduce the queue length and create a systematic process for bulk service. Boer et al. [3] have presented in this article aims to make a meaningful contribution to the domain of operations and production management through a comprehensive exploration of theoretical perspectives. By delving into the intricate interplay of concepts, methodologies, and practices, the study endeavours to enrich the existing body of knowledge in the field. Through a meticulous analysis of established theories, emerging trends, and industry best practices, this research seeks to identify gaps and opportunities for theoretical advancements. By bridging these gaps, the study aims to refine and expand the theoretical foundations that underpin operations and production management. Furthermore, the article employs empirical evidence and case studies to substantiate its theoretical propositions. By grounding theoretical insights in real-world scenarios, the research not only enhances the academic discourse but also offers practical implications for industry professionals. Chen et al. [4] explored the integration of queuing theory and simulated annealing techniques to optimize facility layout design within an Automated Guided Vehicle (AGV)-based modular manufacturing system. The study addresses the complex challenge of arranging workstations and paths to enhance system efficiency and minimize material handling delays. Queuing theory is employed to model the interactions between AGVs and workstations, facilitating the identification of bottlenecks and congestion points. Simulated annealing, a metaheuristic optimization approach, is then applied to reconfigure the facility layout iteratively. This process aims to reach an optimal arrangement that reduces travel distances, minimizes production lead times, and enhances overall productivity. The combination of queuing theory and simulated annealing offers a comprehensive methodology to design a facility layout that accounts for both spatial considerations and operational dynamics. Daigle [5] has provided a systematic way to model, study, and optimize the flow of entities (such as data packets in packet telecommunication) as they wait to be processed or served by limited resources. In the context of packet telecommunication, queueing theory allows us to understand how packets are queued up before being transmitted through network nodes, like routers and switches. This theory takes into account factors such as arrival rates of packets, service rates of network elements, and the number of queues available. By applying queueing theory, we can predict key performance metrics like packet delay, queue length, and system throughput. These insights aid in designing efficient and responsive telecommunication systems, optimizing network configurations, and ensuring reliable data transmission. Koskela et al. [6] have worked on effective project management encompasses a multidisciplinary approach that draws insights from both economic and production theories. This paper explores the dynamic interplay between these two domains and their application within the context of project management. Economic theories provide the foundation for evaluating project viability, analysing costs and benefits, and assessing financial risks. They guide decision-makers in making informed choices about resource allocation, funding sources, and project profitability. On the other hand, production theories contribute to optimizing project execution, focusing on streamlining processes, maximizing resource utilization, and enhancing operational efficiency. By integrating elements from both economic and production theories, project managers can strike a balance between financial considerations and operational excellence. Pasandideh et al. [7] have worked on Genetic application in a facility location problem with random demand within queuing framework. Rece et al. [8] have delved into the application of queueing theory-based mathematical models to the optimization of enterprise organization and industrial production processes. Queueing theory, a fundamental branch of operations research, provides a systematic framework to analyse the behaviour of queues or waiting lines, prevalent in a wide array of real-world systems. By integrating queueing theory into enterprise organization, this research offers insights into efficient resource allocation, workforce management, and customer service enhancement. Through mathematical modelling, the study explores the effects of different queueing strategies on customer satisfaction, waiting times, and operational costs, providing decision-makers with tools to make informed choices. The application of queueing theory in industrial production offers a means to optimize manufacturing processes, minimize bottlenecks, and streamline production flows. The study investigates how various queueing configurations impact production efficiency, lead times, and resource utilization. Such insights facilitate the design of leaner and more agile production systems. Sharma et al. [9] have worked on queueing theory approach with queueing model. This study provides some basic structure for multi-dimensional research. Thurer et al. [10] have worked on three decades of workload control research. This research delves into an extensive examination of research conducted over the span of thirty years in the realm of workload control. This systematic review encompasses a comprehensive analysis of scholarly works related to workload control, aiming to provide a holistic understanding of the advancements, trends, and developments in this field. Workload control refers to the management and coordination of tasks, activities, and resources within a production or operational environment. Tian et al. [11] have worked on vacation queueing models. They provide the theory and applications which is very useful for further research. Ulku et al. [12] have conducted experiments shed light on the intricate relationship between queueing experiences and consumer behaviour. The findings underscore the notion that queues, often regarded as tedious and negative aspects of service encounters, can be leveraged to enhance overall consumer satisfaction and subsequent consumption patterns. The results reveal that certain types of queueing experiences, particularly those infused with entertainment or personalized interactions, have the potential to positively influence consumers' perceptions and attitudes. These enriched queueing experiences contribute to a sense of anticipation and engagement, ultimately translating into increased willingness to spend and higher levels of satisfaction. it is crucial to recognize that not all queueing situations produce similar effects. Factors such as queue length, context, and customer preferences play pivotal roles in shaping the impact of queueing on consumption behaviour. As businesses seek to optimize customer experiences, understanding the nuances of these factors becomes paramount. Vorholter et al. [13] have worked on mathematical modelling. They provided some important model for exploring queueing theory. Yanjun et al. [14] have researched on mathematical modelling methods and their application in the analysis of Complex Signal Systems. This paper aims to explore the pivotal role of queueing theory in manufacturing by examining its applications, methodologies, and real-world impact. By analysing arrival rates, service times, and system capacities, queueing theory enables the identification of operational bottlenecks and the formulation of strategies for more efficient workflows as shown in Figure 1. Additionally, it facilitates the assessment of trade-offs between factors like waiting times, resource utilization, and production costs, enabling manufacturers to make informed decisions. The integration of queueing theory principles into manufacturing practices offers several tangible benefits. These include reduced production downtime, improved customer satisfaction through timely deliveries, better resource allocation leading to cost savings, and enhanced decision-making based on quantitative insights. As manufacturing processes continue to evolve with technological advancements and increasing demand variability, queueing theory provides a robust foundation for addressing the challenges inherent in such environments.

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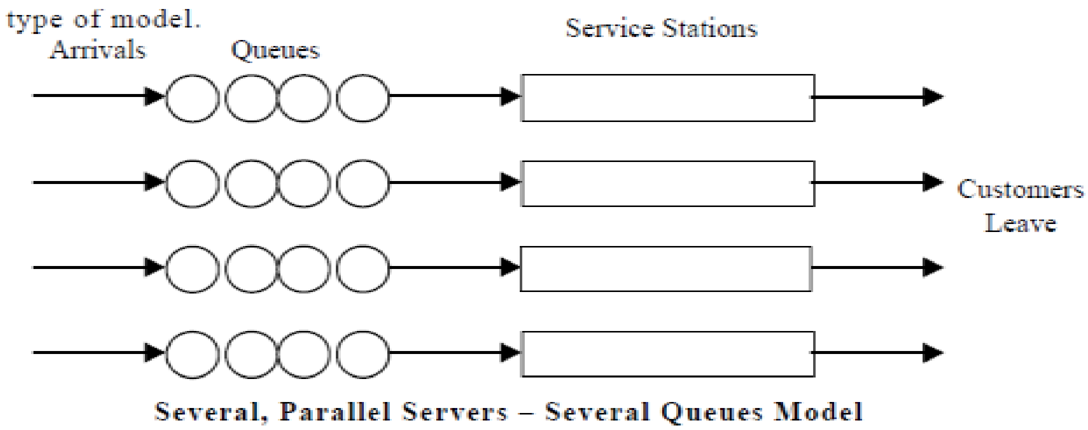

Figure 1 **Queueing System Process**

In the subsequent sections of this paper, we will delve into the key concepts of queueing theory, its application in various manufacturing scenarios, and case studies showcasing its successful implementation. By shedding light on these aspects, we hope to underscore the transformative potential of queueing theory in revolutionizing manufacturing practices and contributing to operational excellence.

2. Need of the Study

The modern manufacturing landscape is characterized by intricate workflows, dynamic demand patterns, and complex interactions among various production stages. In such a context, efficient resource utilization, minimized wait times, and streamlined processes are paramount for maintaining competitiveness and ensuring customer satisfaction. Queueing theory emerges as a vital tool to address these challenges and optimize manufacturing operations.

2.1 Efficiency Enhancement

Manufacturing systems often suffer from inefficiencies due to unbalanced workloads, equipment downtime, and suboptimal resource allocation. Queueing theory's application can uncover hidden bottlenecks and identify areas where resources are underutilized, thus paving the way for more streamlined and efficient processes.

2.2 Wait Time Reduction

Excessive wait times, whether for processing or material movement, can lead to increased lead times, delayed deliveries, and dissatisfied customers. Queueing theory provides insights into reducing these wait times by strategically allocating resources, optimizing production schedules, and managing work-in-progress effectively.

2.3 Resource Allocation Optimization

Manufacturing operations involve the allocation of limited resources such as machines, labour, and materials. Queueing theory's mathematical models enable manufacturers to make data-driven decisions about how to allocate these resources to achieve maximum throughput and minimal idle time.

2.4 Cost Savings

Inefficient processes often lead to higher operational costs due to overstaffing, overtime expenses, and excess inventory. By analysing queues and optimizing system parameters, queueing theory can help manufacturers cut down on unnecessary expenses and achieve cost-effective production.

2.5 Capacity Planning

Manufacturers need to anticipate fluctuations in demand and ensure that their systems can handle peak loads without compromising efficiency. Queueing theory aids in capacity planning by simulating scenarios, predicting resource requirements, and allowing manufacturers to proactively adjust their operations.

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2.6 Decision-Making Support

In an environment with multiple variables and trade-offs, informed decision-making is critical. Queueing theory's quantitative models provide a foundation for making strategic choices related to production processes, resource investments, and system improvements.

2.7 Technology Integration

With the rise of Industry 4.0 technologies, data-driven insights are becoming increasingly crucial. Queueing theory can be integrated with data analytics and real-time monitoring systems to provide continuous feedback for process improvement and adaptive decision-making.

2.8 Continuous Improvement

Manufacturing is an ever-evolving field, and optimizing processes is an ongoing endeavor. Queueing theory provides a structured approach for continuous improvement by allowing manufacturers to monitor, analyse, and adjust their operations based on changing conditions.

The study of queueing theory's role in manufacturing is imperative to address the complexities and challenges posed by modern production environments. By harnessing its principles, manufacturers can optimize resource allocation, reduce wait times, enhance efficiency, and ultimately deliver higher-quality products to meet customer expectations in a competitive market.

3. Assumptions and Notations

3.1 Assumptions

The following assumptions are used in the proposed work:

(i) To calculate the available resources, characterize the demand and manpower planning in Manufacturing system using queueing theory.

(ii) Analysis and management of manufacturing system with modification of service quality using some queueing techniques.

(iii)To explore the role of Queueing Theory in manufacturing and its significance in optimizing complex production systems.

3.2 Notations

Let's define the following notations:

λ	:	Mean arrival rate of costumers
μ	:	Mean service rate
λ_n	:	Mean arrival rate when n Patients are present in the system
μ_n	:	Mean service rate when n Patients are present in the system
ρ	:	Traffic intensity, i.e. $\rho = \frac{\lambda}{S\mu}$
$P_n(t)$	:	Probability when n units are present in the system at time t.
L	:	Expected number of Costumers in the system.
L_q	:	Expected number of Costumers in the Queue.
W	:	Waiting time in the system (with service time).
W_q	:	Waiting time in the queue (without service time).

4. Queueing theory Uses in Manufacturing Sector

Queueing theory plays a crucial role in analysing and optimizing manufacturing processes, where queues of jobs or products waiting for processing can significantly impact efficiency and resource utilization. It provides useful data for decision making process. Queuing theory principles allow manufacturing system administrators to calculate important performance measures, such as the average waiting time, the average number of customers in the queue, and the utilization of available resources. Let's consider a manufacturing scenario with multiple machines and a focus on optimizing the system's performance using queueing theory.

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4.1 The {M/M/S: ∞/FCFS} Queueing model

The {M/M/S: ∞/FCFS} Queueing model is a widely used queuing model to manage manufacturing sector. In this system, the costumer arrival times follow a Poisson distribution and the service time follows the exponential distribution. The number of servers is "S". System capacity is infinite and the service discipline is "First Come, First Serve".

The mean service rate

$$\mu_n = n\mu, \quad for\ 0 \le n \le S$$
$$= S\mu, \quad for\ n \ge S$$

Let n costumers are present in the system in time t and $n \ge S$.

Then the study state equations are

$$-\lambda P_0 + \mu P_1 = 0 \tag{1}$$

$$\lambda P_{n-1} - (\lambda + n\mu)P_n + (n+1)\mu P_{n+1} = 0; n \le S-1 \tag{2}$$

$$\lambda P_{n-1} - (\lambda + S\mu)P_n + S\mu P_{n+1} = 0 \quad ; n \ge S \tag{3}$$

To solve study state equations, we get

$$P_n = \begin{bmatrix} \dfrac{(n\rho)^n}{\lfloor n} P_0, 1 \le n \le S \\ \dfrac{S^S \rho^n}{\lfloor S} P_0, n \ge S \end{bmatrix} \tag{4}$$

And,

$$P_0 = \frac{1}{\left[\sum_{n=0}^{S-1} \dfrac{(S\rho)^n}{\lfloor n} + \dfrac{(S\rho)^S}{(1-\rho)\lfloor S}\right]}, \text{ Where } \rho = \frac{\lambda}{\mu S} \tag{5}$$

From these equations (1) to (5), we can find out the probability of arrived costumers or serviced costumers at the specified manufacturing system. Further, we find out the key performance measures related to this queueing system, which are as follows:

(a) To find Expected number of Costumers in the Queue

$$L_q = \frac{\rho P_S}{(1-\rho)^2}, \text{ where } P_S \text{ is given by eq. (5)} \tag{6}$$

(b) To find expected number of Costumers in the system

$$L = \frac{\rho P_S}{(1-\rho)^2} + S\rho \tag{7}$$

(c) To find expected waiting time per Costumers in the system (with service time)

$$W = \frac{\rho P_S}{\lambda(1-\rho)^2} + \frac{S\rho}{\lambda} \tag{8}$$

(d) To find expected waiting time per Costumers in the queue (without service time)

$$W_q = \frac{\rho P_S}{\lambda(1-\rho)^2} \tag{9}$$

From these equations (6) to (9), we can calculate the average number of costumers waiting and getting treatment at the same time, Expected number of Costumers in the system, Expected waiting time per Costumers in the system (with service time) and Expected waiting time per Costumers in the queue (without service time). These calculations provide us the valuable data for decision making in Manufacturing System. Queueing theory offers tools to analyse more intricate scenarios involving multiple machines, different job types, and more complex routing of jobs through the manufacturing process. The proof of the queueing model is not given here. The interested reader will find proof of the above formulas in the works of Hiller and Lieberman [15]. Now we describe a graph of comparative study between Expected number of Costumers in the system (L) and Utilization factor (ρ) for various number of servers (S). Figure 2. Shows how L changes with ρ for various values of S.

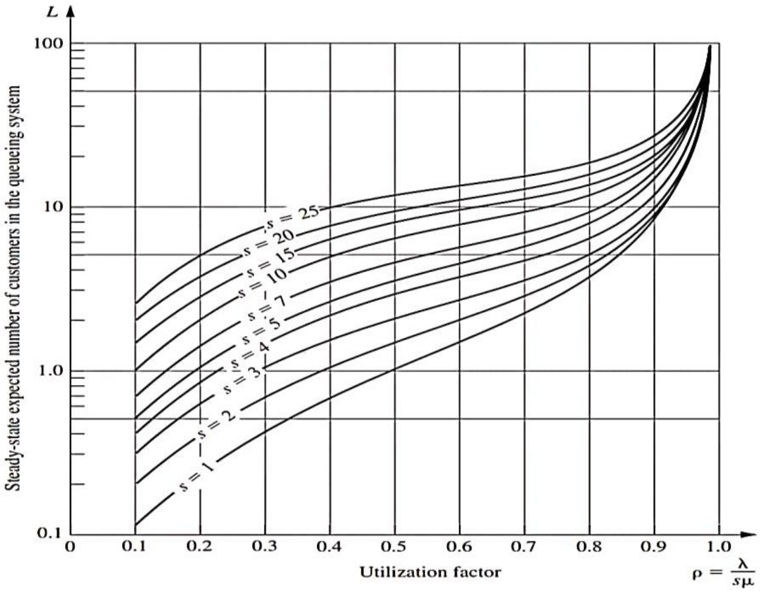

Figure 2 **Values of L for M/M/S model for various values of S**

4.2 Scenario: Single Machine Manufacturing System

In this scenario, we have a manufacturing system with a single machine that processes jobs as they arrive. The jobs are characterized by their arrival rate (λ) and the time it takes for the machine to process each job (1/μ). The goal is to analyse the system's performance using queueing theory and optimize key performance measures.

For S=1, The Key Performance Measures are:

Average Queue Length (L_q): The average number of jobs waiting in the queue.

The average queue length is:

$$L_q = \frac{\lambda^2}{\mu(\mu - \lambda)}$$

Average Number of Customers in the System (L): The average number of jobs waiting in the system is,

$$L = \frac{\lambda}{\mu - \lambda}$$

Average Waiting Time (W_q): The average time a job spends waiting in the queue before being processed. The average waiting time in the queue can be calculated using Little's Law

$$W_q = \frac{L_q}{\lambda} = \frac{\lambda}{\mu(\mu - \lambda)}$$

Average Waiting Time in the System (W): The average waiting time in the system can be calculated as

$$W = \frac{1}{\mu - \lambda}$$

System Utilization (ρ): The system utilization (server utilization) is given by the ratio of arrival rate to service rate

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$$\rho = \frac{\lambda}{\mu}$$

Throughput (X): Throughput represents the rate at which customers are being served $X = \lambda$

Example:
Let's consider the following data for our manufacturing system:
Arrival Rate (λ) = 10 jobs per hour
Service Rate (μ) = 15 jobs per hour
Calculations:
Utilization Factor (ρ): $\rho = \frac{\lambda}{\mu} = \frac{10}{15} = \frac{2}{3} < 1$

Average Queue Length (L_q): $Lq = \frac{\lambda^2}{\mu(\mu-\lambda)} = \frac{10^2}{15(15-10)} = \frac{100}{75} = \frac{4}{3} \text{ or } 1.33$

Average Waiting Time (W_q): $Wq = \frac{Lq}{\lambda} = \frac{4/3}{10}$ = 0.133 hours or 8 minutes

Explanation:
In this scenario, the utilization factor indicates that the machine is being used efficiently, as it's busy 2/3 of the time. The average queue length of 1.33 suggests that on average, there are about 1.33 jobs waiting in the queue to be processed. The average waiting time of 8 minutes indicates that jobs spend, on average, 8 minutes waiting in the queue before being processed. These performance measures provide insights into the system's efficiency and can guide to optimize the manufacturing process, such as adjusting arrival rates, the number of machines or servers.

5. System Analysis

Queueing theory plays a key role in optimizing manufacturing processes by providing insights into resource allocation, system efficiency, and overall productivity. In manufacturing, queues often form at different stages, such as machine setups, inspections, and material handling. By applying queueing models, manufacturers can determine optimal buffer sizes, staffing levels, and scheduling strategies to minimize wait times and maximize throughput. This theory aids in understanding the trade-offs between resource utilization and customer service levels, enabling informed decisions on process improvements. Additionally, queueing theory facilitates the identification of bottlenecks, ensuring smoother production flows and reduced lead times. As manufacturing systems become more complex, queueing theory continues to guide effective resource management, ensuring that production processes remain streamlined and efficient, ultimately contributing to enhanced operational performance and customer satisfaction. Here's how Queueing Theory is applied in manufacturing, along with illustrative tables:

5.1 Process Modelling and Analysis

Queueing Theory assists in modelling and analysing manufacturing processes with multiple workstations or stages. This enables a better understanding of system behavior and helps identify areas for improvement.

Table 1 Overview of Processing Time and Service Rate

Stage	Processing Time (mean)	Service Rate (1/mean)
A	10 units/min	0.1 min/unit
B	15 units/min	0.067 min/unit
C	8 units/min	0.125 min/unit

The Table 1 provides an overview of a manufacturing process with three distinct stages- designated as A, B, and C, by presenting the key parameters of processing time and service rate for each stage. These parameters are fundamental in understanding the speed and efficiency of each

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stage's operations within the larger manufacturing process. In stage A, the average processing time for a unit is 10 minutes, resulting in a service rate of 0.1 units per minute, which can also be expressed as 0.1 minutes per unit. This rate signifies that stage A can process 0.1 units in a minute, translating to 6 units every hour. The reciprocal of the processing time gives the service rate, and it provides insight into the speed at which units are being worked on or processed within the stage. For stage B, the average processing time is 15 minutes, yielding a service rate of approximately 0.067 units per minute, equivalent to 4 units per hour. This indicates that stage B operates at a slightly slower pace than stage A due to the longer processing time. In stage C, the average processing time is 8 minutes, leading to a service rate of 0.125 units per minute or 7.5 units per hour. This suggests that stage C operates more efficiently in terms of processing speed compared to stage B, despite having a shorter processing time than stage A.

5.2 Bottleneck Identification

Queueing models highlight bottlenecks – stages where the queue length exceeds capacity, leading to delays. Addressing bottlenecks optimizes the overall throughput.

Table 2 Overview of Processing Time, Service Rate and Utilization

Stage	Processing Time (mean)	Service Rate (1/mean)	Utilization (ρ)
A	9 units/min	10 units/min	0.9
B	8 units/min	15 units/min	0.533
C	7 units/min	8 units/min	0.875

The Table 2 presents a comprehensive view of a manufacturing process involving three distinct stages - designated as A, B, and C, by providing key metrics related to processing time, service rate, and utilization. These metrics are critical in assessing the efficiency and performance of each stage within the larger manufacturing process. In stage A, the average processing time for a unit is 9 minutes, which corresponds to a service rate of 10 units per minute (since service rate is the reciprocal of processing time). The utilization (ρ), which is the ratio of the average arrival rate to the service rate, is calculated to be 0.9. This implies that the stage is operating at a high utilization rate, with demand approaching the capacity of the system. Such high utilization could potentially lead to increased queue lengths and wait times, indicating a need for careful resource management to prevent congestion. For stage B, the average processing time is 8 minutes, resulting in a service rate of 15 units per minute. The utilization rate is calculated to be 0.533. This indicates that the stage is operating at a relatively moderate utilization level, suggesting a balanced situation where the capacity can accommodate the incoming demand with some room for fluctuations. In stage C, the average processing time is 7 minutes, leading to a service rate of 8 units per minute. The calculated utilization is 0.875, which signifies that the stage is operating at a high utilization rate. Similar to stage A, this high utilization level could potentially result in increased queue lengths and wait times.

5.3 Queue Length and Waiting Time Analysis

Queueing Theory calculates average queue lengths and wait times, aiding in resource allocation and scheduling decisions.

Table 3 Performance analysis using Queueing Theory Principles

Stage	Arrival Rate (λ)	Service Rate (μ)	Service Rate (μ)	Average Wait Time (W_q)
A	9 units/min	10 units/min	4.05	0.45 min
B	8 units/min	15 units/min	0.23	0.03 min
C	7 units/min	8 units/min	2.625	0.375 min

The Table 3 provides an insight into the performance of a manufacturing process involving three stages, denoted as A, B, and C, using Queueing Theory principles. Arrival rate (λ) represents the average rate at which units arrive at each stage, while the service rate (μ) signifies the average rate at which units are processed. Additionally, the average wait time (W_q) is the time units spend waiting in the queue before being processed. In stage A, units arrive at a rate of 9 units per minute, slightly exceeding the processing capacity of 10 units per minute. This results in an average queue length of 4.05 units and an average wait time of 0.45 minutes. The stage's service rate is higher than its arrival rate, but due to occasional spikes in arrivals, some units end up waiting. In stage B, units arrive at a rate of 8 units per minute, while the processing capacity is 15 units per minute. The stage operates comfortably with an average queue length of 0.23 units and an average wait time of 0.03 minutes. The service rate exceeds the arrival rate, leading to minimal waiting times and efficient processing. In stage C, units arrive at a rate of 7 units per minute, and the processing capacity is 8 units per minute. The average queue length is 2.625 units, and the average wait time is 0.375 minutes. The arrival rate is close to the service rate, resulting in moderate queue lengths and wait times.

Conclusion

The exploration of Queueing Theory's role in manufacturing underscores its significance in optimizing complex production systems. This study has illuminated how Queueing Theory offers valuable insights into resource allocation, wait time reduction, and overall efficiency enhancement. In this study, we describe a M/M/S queueing model with its key performance measures. Further we analyse these results for Single Machine Manufacturing System. Here we also analyse manufacturing system with Process Modelling, Bottleneck Identification, Queue length and Waiting time analysis. By uncovering hidden bottlenecks, informing decision-making, and guiding capacity planning, Queueing Theory becomes a foundational tool for continuous improvement in manufacturing processes.

Future Scope

As modern manufacturing evolves with technological advancements and dynamic demand patterns, Queueing Theory's quantitative approach provides a systematic way to address challenges and seize opportunities. Its integration with data analytics and Industry 4.0 technologies further enhances its relevance. Through its ability to streamline workflows, minimize costs, and enhance customer satisfaction, Queueing Theory emerges as a driving force behind operational excellence in manufacturing, contributing to sustained competitiveness and growth in an ever-changing landscape.

Social Impact of the Proposed Work

The impact and utility of this proposed work are as follows:

1. This work will analyse that how Queueing Theory offers valuable insights into resource allocation, wait time reduction, and overall efficiency enhancement.
2. This work will provide model and algorithms for the decision support systems by calculating the arrival rate of costumers and service capacity.
3. This work will provide a base for decision support system with valuable data and methodology.
4. This work will provide optimization model of queueing with quality service guarantee in management.
5. This work will lead a valuable contribution in solving realistic problems and help to the policymakers and planners to make better decisions.

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