International Research Journal on Advanced Engineering and Management
https://goldncloudpublications.com
https://doi.org/10.47392/IRJAEM.2024.501

e ISSN: 2584-2854
Volume: 02
Issue:11 November 2024
Page No: 3405-3410

Fuzzy Quadripartitioned Neutrosophic Matrices in Strategy Finding

P.Sheeba Maybell[1], *S.Indhumathi*[2]
[1,2]*Assistant Professor, Dept. of Mathematics, Nehru Arts and Science College., Coimbatore, Tamilnadu, India.*
Email ID: *nascsheeba.maybell@nehrucolleges.com*[1], *nascsindhumathimaths@nehrucolleges.com*[2]

Abstract
Researchers have primarily proposed fuzzy oriented matrix theories have to model uncertainty for real-world decision-making (DM). Fuzzy neutrosophic soft matrix (FNSM) theory has been developed recently by combining the fuzzy neutrosophic set and soft set to handle the indeterminate information parametrically. The concept of fuzzy Quadripartitioned neutrosophic soft matrices are discussed. Some properties of trace, Determinant of a Fuzzy Quadripartitioned Neutrosophic Soft Matrix with some examples were discussed. An application of FQNSMS for Dimensionality reduction in data mining were discussed with an algorithm.

Keywords: *Soft set; quadripartitioned neutrosophic soft set; fuzzy quadripartitioned neutrosophic soft set; fuzzy quadripartitioned neutrosophic soft matrix; decision-making.*

1. Introduction

The concept of fuzzy set was introduced by Zadeh in 1965. In many real applications to uncertainty, where $\mu_A(x) \in [0,1]$ is used to represent the grade of membership of a fuzzy set . Attanasov in 1986 introduced Intuitionistic Fuzzy set (IFS) which handle incomplete information with the grade of membership function, non-membership function and indeterminate function. Smarandache in 1995 introduced another concept of imprecise data called neutrosophic data. Neutrosophic data were developed to deal with complicating aspects to process imprecision, vagueness, and uncertainty in data. It is also a generalisation of Intuitionistic fuzzy sets. The concept of Soft theory was formulated by Molodstov in 1999 and Maji initiated the idea of Fuzzy Soft set. Chatterjee initiated the notion of Quadripartitioned single valued neutrosophic set (QSVNS). The QSVNs provide more ideas of the indeterminacy component which is divided into two parts as Contradiction and Ignorance value. Sumathi and Arockiarani initiated a new operation on Fuzzy Neutrosophic Soft Matrices (FNSMs). Kavitha et.al, presented the idea of unique solvability of max-min operation through FNSM and the T- Ordering and minus ordering of FNSMs. Uma et.al, presented two methods of fuzzy neutrosophic soft matrices. The concept of Fuzzy Neutrosophic set (FNS) and Quadripartitioned Neutrosophic set were combined and evolved as Fuzzy Quadripartitioned Neutrosophic set (FQNS) and this was first proposed by Somnath Debnath. Fuzzy quadripartitioned Neutrosophic set (FQNS) was embedded with soft set and formed as Fuzzy quadripartitioned Neutrosophic soft set (FQNSS). A new matrix theory called a Fuzzy quadripartitioned Neutrosophic soft matrix (FQNSM) theory is established by Somnath Debnath and in this work some properties with DM were discussed.

2. Literature Review

Kim and Roush [9] developed the theory of fuzzy matrices analogous to that of Boolean matrices. Kim and Baartmans [10] also developed the concept of determinant theory for fuzzy matrices. Jian mia chen [3] introduced the fuzzy matrix partial ordering and generalised inverse. Basic properties of Intuitionistic fuzzy matrices as a generalization of the results on fuzzy matrices have been derived by Khan & Pal [15].Soft set theory was introduced by Molodtsov [4] for modelling vagueness and uncertainty. Yang and Ji [25] introduced a matrix representation of fuzzy soft set and to apply in decision making. Borah, Neog, Sut [20] introduced fuzzy soft matrix and its decision making. Smarandache [5] introduced the concept of Neutrosophic set to handle problems involving imprecise, indeterminacy and inconsistent data. Maji [6] initiated the the neutrosophic soft set. Deli and Broumi [14] introduced neutrosophic soft matrices and utilize them as NSM decision making. Sumathi and Arockiarani [12,13] introduced new operation on

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fuzzy neutrosophic topological spaces and they established into a new theory for Fuzzy neutrosophic soft matrices. Uma, Sriram & Murugadas [22] gave us fuzzy neutrosophic soft matrices methods of 2-types and their operations were discussed. They also discussed about the determinant and adjoint of fuzzy neutrosophic soft matrices[16]. Kavitha, Murugadas and Sriram have introduced the concept of T-ordering and Minus ordering of FNSMs was discussed by them [17, 18]. A new concept of Quadripartitioned single valued Neutrosophic set (QSVNS) was proposed by Chatterjee [19]. The QSVNS technique was further extended by Somen Debnath [7] as a new set of Fuzzy quadripartitioned neutrosophic set (FQNS).The original extension is made by Somen Debnath when soft set is imbedded in this concept as fuzzy quadripartitioned neutrosophic soft set and fuzzy quadripartitioned neutrosophic soft matrices and their properties were discussed.

3. Neutrosophic Set

Let X be a universe of discourse, with a generic element in X denoted by x, the neutrosophic set (NS), A is an object having the structure.

$$A = \{< x, (T_A(x), I_A(x), F_A(x)>, x \in X\}$$

Where the function defines $T, I, F: X \rightarrow]^{-}0, 1^{+}[$ respectively the degree of membership (or truth), the degree of indeterminacy and the degree of non-membership (or Falsehood) of the component $x \in X$ to the set A with the property

$$^{-}0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3^{+} \ldots\ldots\ldots(1).$$

We assume the value from the subset of [0,1]. we read equation (1) as $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$ In brief an component $\tilde{a}$ in the neutrosophic set A, can be read as $\tilde{a} = <a^T, a^I, a^F>$, where a^T symbolise the level of Truth, a^I symbolise the level of Indeterminacy, a^F symbolise the level of Falsity, such that $0 \leq a^T + a^I + a^F \leq 3$.

3.1. Quadripartitioned Neutrosophic Set (QNS)

A quadripartitioned neutrosophic set S over the set of the universe X is defined as

$$S = \{< x, (T_S(x), C_S(x), U_S(x), F_S(x)>, x \in X\}$$

where the function defines $T, C, U, F: X \rightarrow]^{-}0, 1^{+}[$ respectively the degree of truth membership, the degree of contradiction membership, the degree of ignorance membership and the degree of Falsehood membership (or) of the component $x \in X$ to the set A with the property $^{-}0 \leq T_S(x) + C_S(x) + U_S(x) + F_S(x) \leq 4^{+} \ldots\ldots\ldots(1)$ We assume the value from the subset of [0,1]. we read equation (1) as

$$0 \leq T_S(x) + C_S(x) + U_S(x) + F_S(x) \leq 4$$

3.2. Definition

[7] The Fuzzy Quadripartitioned neutrosophic set K over the universe of discourse X is defined as

$$K = \{< u, (T_S(u), C_S(u), U_S(u), F_S(u)>, u \in X\}$$

where $T_k, C_k, F_k, I_k: X \rightarrow [0,1]$ such that $0 \leq T_S(u) + C_S(u) + U_S(u) + F_S(u) \leq 4$.

3.3. Definition

Let U be the initial universal set and E be a set of parameter. Consider a non-empty set A, $A \subset E$. The family (F, A) is named as the Fuzzy Neutrosophic Soft sets (FNSS) over U, where F is a mapping given by $F: A \rightarrow P(U)$. Let $P(U)$ signifies the set of all fuzzy neutrosophic sets of U. We assume A as FNSS over U rather than (F, A).

3.4. Definition

[7] Let $\vec{X} = \{\overrightarrow{u_1}, \overrightarrow{u_2}, \overrightarrow{u_3}, \ldots\ldots \overrightarrow{u_m}\}$ be the universal set and $\vec{E}$ be the set of parameter given by $\vec{E} = \{\overrightarrow{\varepsilon_1}, \overrightarrow{\varepsilon_2}, \overrightarrow{\varepsilon_3} \ldots\ldots \overrightarrow{\varepsilon_n}\}$. For $\vec{B} \subseteq \vec{E}$. A pair $(\vec{G}, I^{FQNS\vec{X}})$ be a FQNSS over $\vec{X}$. Then the relation set $\mathfrak{R}_{\vec{B}}$ can be considered as a subset of the cartesian product $\vec{X} \times \vec{E}$ is defined by

$$\mathfrak{R}_{\vec{B}} = \{(\vec{u}, \vec{\varepsilon}); \vec{\varepsilon} \in \vec{B}, \vec{u} \in \lambda_{\vec{B}}(\vec{\varepsilon})\}$$

which is called a relation form of $(\lambda_{\vec{B}}, \vec{E})$. Based on the relation set $R_{\vec{B}}$ we define $T_{\mathfrak{R}_{\vec{B}}}, C_{\mathfrak{R}_{\vec{B}}}, F_{\mathfrak{R}_{\vec{B}}}, I_{\mathfrak{R}_{\vec{B}}}: \vec{X} \times \vec{E} \rightarrow [0,1]$ as the truth, contradiction, ignorance and falsity membership functions respectively and they used to obtain the respective Membership values of $\vec{u} \in \vec{X}$ for each $\vec{\varepsilon} \in \vec{E}$. Notation: In this work we notate FQNSM as If

$$\vec{A}_{m \times n} = [< \vec{a}_{ij}^T(\vec{u}_i, \vec{\varepsilon}_j), \vec{a}_{ij}^C(\vec{u}_i, \vec{\varepsilon}_j), \vec{a}_{ij}^U(\vec{u}_i, \vec{\varepsilon}_j), \vec{a}_{ij}^F(\vec{u}_i, \vec{\varepsilon}_j) >]$$

This matrix is called an m x n Fuzzy Quadripartitioned Neutrosophic Soft Matrix (FQNSM) of the FQNSS $(\lambda_{\vec{B}}, \vec{E})$ over $\vec{X}$ whose order is m over n.

3.5. Definition

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[7] Let $(\vec{G}, I^{FQNS\vec{X}})$ be a FQNSS over $\vec{X}$,where $\vec{G}: \vec{B} \rightarrow I^{FQNS\vec{X}}$ .Then the matrix representation of $(\vec{G}, I^{FQNS\vec{X}})$ is $\vec{B}_{m\times n} = [\vec{b}_{ij}]_{m\times n}$ and it is defined as

$$\vec{b}_{ij} = \begin{cases} (< a_{T_j}(\vec{B}_i), a_{C_j}(\vec{B}_i), a_{U_j}(\vec{B}_i), a_{F_j}(\vec{B}_i) >) & if\ \vec{\varepsilon_i} \in \vec{B} \\ (< 0, 0, 1, 1 >) & if\ \vec{\varepsilon_i} \notin \vec{B} \end{cases}$$

3.6. Definition [7]

A FQNSM of any order is said to be a fuzzy quadripartitioned neutrosophic soft null matrix if all its elements are <0, 0, 1, 1> and it is denoted by $\vec{\Phi}$

3.7. Definition [7]

A FQNSM of any order is said to be a FQNS universal matrix if all its elements are < 1, 1 ,0, 0> and it is denoted by $\vec{J}$

3.8. Definition [7]

If $\vec{A} = [< \vec{a}_{ij}^T, \vec{a}_{ij}^C, \vec{a}_{ij}^U, \vec{a}_{ij}^F >]_{m\times n}$ be a FQNSM then $\vec{A}^c$ is denoted as fuzzy quadripartitioned neutrosophic soft compliment matrix is

$$\vec{A}^c = [< 1 - \vec{a}_{ij}^T, 1- \vec{a}_{ij}^C, 1- \vec{a}_{ij}^U, 1- \vec{a}_{ij}^F >]_{m\times n}$$

3.9. Definition

If $\vec{A} = [< \vec{a}_{ij}^T, \vec{a}_{ij}^C, \vec{a}_{ij}^U, \vec{a}_{ij}^F >]_{m\times n}$ be a FQNSM then $\vec{A}^t$ is denoted as transpose of a FQNSM then

$$\vec{A}^t = [< \vec{a}_{ji}^T, \vec{a}_{ji}^C, \vec{a}_{ji}^U, \vec{a}_{ji}^F >]_{m\times n}$$

3.10. Definition

Let $\vec{A}$ be a square FQNSM then the trace of a FQNSM is symbolised by tr($\vec{A}$) is the maximisation of the leading diagonal elements. Then,

$$tr(\vec{A}) = [< \max(\vec{a}_{ii}^T), \max(\vec{a}_{ii}^C), \min(\vec{a}_{ii}^U), \min(\vec{a}_{ii}^F) >]$$

3.11. Definition

Let $\vec{A} = [< \vec{a}_{ij}^T, \vec{a}_{ij}^C, \vec{a}_{ij}^U, \vec{a}_{ij}^F >]_{m\times n}$ be a FQNSM and $k \in$ F =[0,1] define scalar multiplication as

$$k\vec{A} = [\min(k, \vec{a}_{ij}^T), \min(k, \vec{a}_{ij}^C), \max(k, \vec{a}_{ij}^U), \max(k, \vec{a}_{ij}^F)] \in FNQSM_{m \times n}$$

3.12. Definition

Let $\vec{A} = [< \vec{a}_{ij}^T, \vec{a}_{ij}^C, \vec{a}_{ij}^U, \vec{a}_{ij}^F >]_{m\times n}$ and $\vec{B}$=[$< \vec{b}_{ij}^T, \vec{b}_{ij}^C, \vec{b}_{ij}^U, \vec{b}_{ij}^F >]_{m\times n}$ be a FQNSM then

$$\vec{A} \cup \vec{B} = <\max(\vec{a}_{ij}^T, \vec{b}_{ij}^T), \max(\vec{a}_{ij}^C, \vec{b}_{ij}^C), \min(\vec{a}_{ij}^U, \vec{b}_{ij}^U), \min(\vec{a}_{ij}^F, \vec{b}_{ij}^F) >$$

$$\vec{A} \cap \vec{B} = \min(\vec{a}_{ij}^T, \vec{b}_{ij}^T), \min(\vec{a}_{ij}^C, \vec{b}_{ij}^C), \max(\vec{a}_{ij}^U, \vec{b}_{ij}^U), \max(\vec{a}_{ij}^F, \vec{b}_{ij}^F) >$$

3.13. Definition

Let $\vec{A} = [< \vec{a}_{ij}^T, \vec{a}_{ij}^C, \vec{a}_{ij}^U, \vec{a}_{ij}^F >]_{m\times p}$ and $\vec{B}$=[$< \vec{b}_{jk}^T, \vec{b}_{jk}^C, \vec{b}_{jk}^U, \vec{b}_{jk}^F >]_{n\times p}$ be a FQNSM then their product is denoted and defined as

$$\vec{A} * \vec{B} = [< \sum_{k=1}^{n}(\vec{a}_{ij}^T \wedge \vec{b}_{jk}^T), \sum_{k=1}^{n}(\vec{a}_{ij}^C \wedge \vec{b}_{jk}^C), \prod_{k=1}^{n}(\vec{a}_{ij}^U \vee \vec{b}_{jk}^U), \prod_{k=1}^{n}(\vec{a}_{ij}^F \vee \vec{b}_{jk}^F) >]_{m\times p} \quad \forall\, i, j, k.$$

$$= [< \vee_{k=1}^{n}(\vec{a}_{ij}^T \wedge \vec{b}_{jk}^T), \vee_{k=1}^{n}(\vec{a}_{ij}^C \wedge \vec{b}_{jk}^C), \wedge_{k=1}^{n}(\vec{a}_{ij}^U \vee \vec{b}_{jk}^U), \wedge_{k=1}^{n}(\vec{a}_{ij}^F \vee \vec{b}_{jk}^F) >] \quad \forall\, i, j, k.$$

The product $\vec{A} * \vec{B}$ is defined if and only if the number of column of $\vec{A}$ is the same as the number of rows of $\vec{B}$. $\vec{A}$ and $\vec{B}$ are said to be conformable for multiplication. Here $\sum_{k=1}^{n}(\vec{a}_{ij}^T \wedge \vec{b}_{jk}^T)$ is called max-min operation and $\prod_{k=1}^{n}(\vec{a}_{ij}^U \vee \vec{b}_{jk}^U)$ is called min-max operation.

4. Properties

1. $[\vec{A}^c]^c = \vec{A}$
2. $\vec{A} \cup \vec{B} = \vec{B} \cup \vec{A}$ & $\vec{A} \cap \vec{B} = \vec{B} \cap \vec{A}$
3. $(\vec{A} \cup \vec{B}) \cup \vec{C} = \vec{A} \cup (\vec{B} \cup \vec{C})$ & $(\vec{A} \cap \vec{B}) \cap \vec{C} = \vec{A} \cap (\vec{B} \cap \vec{C})$
4. $(\vec{A} \cap \vec{B}) \cup \vec{C} = (\vec{A} \cup \vec{B}) \cap (\vec{B} \cup \vec{C})$ & $(\vec{A} \cup \vec{B}) \cap \vec{C} = (\vec{A} \cap \vec{B}) \cup (\vec{B} \cap \vec{C})$

The Properties are easily verified.

4.1. Proposition

Let $\vec{A} = [< \vec{a}_{ij}^T, \vec{a}_{ij}^C, \vec{a}_{ij}^U, \vec{a}_{ij}^F >]_{n\times n}$ be a FQNSM then k is a scalar where $0 \le k \le 1$. Then tr($k\vec{A}$) = k tr ($\vec{A}$). Proof: We have tr($k\vec{A}$)=[< max (min(k,, $\vec{a}_{ii}^T$)) , max (min(k,, $\vec{a}_{ii}^C$)), min (max (k, $\vec{a}_{ii}^U$)), min (max (k, $\vec{a}_{ii}^F$)>]

k tr ($\vec{A}$) = (<max k ($\vec{a}_{ii}^T$) , max k ($\vec{a}_{ii}^C$), min k ($\vec{a}_{ii}^U$), min k($\vec{a}_{ii}^F$)>); tr($k\vec{A}$) = k tr ($\vec{A}$)

4.2. Definition

Let $\vec{A} = \{< \vec{a}_{ij}^T, \vec{a}_{ij}^C, \vec{a}_{ij}^U, \vec{a}_{ij}^F >\}_{n\times n}$ be a FQNSM then the determinant

$|\vec{A}|$ =<

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$\vee_{\sigma\epsilon S_n}(\vec{a}^T_{1\sigma(1)} \wedge \ldots \ldots \wedge \vec{a}^T_{n\sigma(n)})$,
$\vee_{\sigma\epsilon S_n}(\vec{a}^C_{1\sigma(1)} \wedge \ldots \wedge \vec{a}^C_{n\sigma(n)})$,
$\wedge_{\sigma\epsilon S_n}(\vec{a}^U_{1\sigma(1)} \vee \ldots \vee \vec{a}^U_{n\sigma(n)})$,
$\wedge_{\sigma\epsilon S_n}(\vec{a}^F_{1\sigma(1)} \vee \ldots \vee \vec{a}^F_{n\sigma(n)}) >$
where S_n denotes the symmetric group of all permutation of the indices (1,2,.......n).

4.3. Example

Let$\vec{A}$=

$$\begin{bmatrix} <(0.5,0.3,\ 0.2,\ 0.4)> & <(0.6,\ 0.7,0.4,\ 0.8)> \\ <(0.9,\ 0.6,\ 0.8,\ 0.7)> & <(0.5,\ 0.7,\ 0.6,\ 0.8)> \end{bmatrix}$$

Then

$|\vec{A}|$=<[(0.5,0.3, 0.2, 0.4) ∧ (0.5 , 0.7, 0.6, 0.8)] ∨ [(0.6, 0.7 , 0.4, 0.8) ∧ (0.9 , 0.6 , 0.8 , 0.7)]>

$|\vec{A}|$ = <[(0.5,0.3, 0.6, 0.8) ∨ (0.6,0.6, 0.8, 0.8)]>

= <[(0.6 , 0.6 , 0.6 , 0.8)]>

4.4. Proposition

If a FQNSM $\vec{B}$ is obtained from an $n \times n$ order FQNSM $\vec{A}$ = $[<\vec{a}^T_{ij}, \vec{a}^C_{ij}, \vec{a}^U_{ij}, \vec{a}^F_{ij}>]_{n\times n}$ by multiplying the ith row of $\vec{A}$ (ith column) by $k \in [0,1]$, then $|\vec{B}| = k\ |\vec{A}|$.Proof: Suppose $\vec{B} = [<\vec{b}^T_{ij}, \vec{b}^C_{ij}, \vec{b}^U_{ij}, \vec{b}^F_{ij}>]_{n\times n}$ then

$|\vec{B}|$=<
$\vee_{\sigma\epsilon S_n}(\vec{b}^T_{1\sigma(1)} \wedge \ldots \ldots \wedge \vec{b}^T_{n\sigma(n),})$,
$\vee_{\sigma\epsilon S_n}(\vec{b}^C_{1\sigma(1)} \wedge \ldots \wedge \vec{b}^C_{n\sigma(n)})$,
$\wedge_{\sigma\epsilon S_n}(\vec{b}^U_{1\sigma(1)} \vee \ldots \vee \vec{b}^U_{n\sigma(n)})$, $\wedge_{\sigma\epsilon S_n}(\vec{b}^F_{1\sigma(1)} \vee \ldots \vee \vec{b}^F_{n\sigma(n)}) >$
=<$\vee_{\sigma\epsilon S_n}(\vec{a}^T_{1\sigma(1)} \wedge \ldots \wedge k\vec{a}^T_{i\sigma(i)} \ldots \wedge \vec{a}^T_{n\sigma(n)})$, $\vee_{\sigma\epsilon S_n}(\overrightarrow{a^C_{1\sigma(1)}} \wedge \ldots \wedge k\vec{a}^C_{i\sigma(i)} \ldots \wedge \vec{a}^C_{n\sigma(n)})$,$\wedge_{\sigma\epsilon S_n}(\vec{a}^U_{1\sigma(1)} \vee \ldots \vee k\vec{a}^U_{i\sigma(i)} \ldots \vee \vec{a}^U_{n\sigma(n)})$, $\wedge_{\sigma\epsilon S_n}(\vec{a}^F_{1\sigma(1)} \vee \ldots \vee k\vec{a}^F_{i\sigma(i)} \ldots \vee \vec{a}^F_{n\sigma(n)}) >$
= $[k\ <\vec{a}^T_{ij}, \vec{a}^C_{ij}, \vec{a}^U_{ij}, \vec{a}^F_{ij}>] = k\ |\vec{A}|$.

4.5. Proposition

Let $\vec{A} = [<\vec{a}^T_{ij}, \vec{a}^C_{ij}, \vec{a}^U_{ij}, \vec{a}^F_{ij}>]_{n\times n}$ be a FQNSM then det ($\vec{A}$) = det ($\vec{A}^t$) where $\vec{A}^t$ denotes the transpose of $\vec{A}$ Proof: Let $\vec{A}\ \epsilon\ FQNSM_{2\times 2}$, then

$$|\vec{A}| = \begin{vmatrix} <\vec{a}^T_{11}, \vec{a}^C_{11}, \vec{a}^U_{11}, \vec{a}^F_{11}> & <\vec{a}^T_{12}, \vec{a}^C_{12}, \vec{a}^U_{12}, \vec{a}^F_{12}> \\ <\vec{a}^T_{21}, \vec{a}^C_{21}, \vec{a}^U_{21}, \vec{a}^F_{21}> & <\vec{a}^T_{22}, \vec{a}^C_{22}, \vec{a}^U_{22}, \vec{a}^F_{22}> \end{vmatrix}$$

$$|\vec{A}^t| = \begin{vmatrix} <\vec{a}^T_{11}, \vec{a}^C_{11}, \vec{a}^U_{11}, \vec{a}^F_{11}> & <\vec{a}^T_{21}, \vec{a}^C_{21}, \vec{a}^U_{21}, \vec{a}^F_{21}> \\ <\vec{a}^T_{12}, \vec{a}^C_{12}, \vec{a}^U_{12}, \vec{a}^F_{12}> & <\vec{a}^T_{22}, \vec{a}^C_{22}, \vec{a}^U_{22}, \vec{a}^F_{22}> \end{vmatrix}$$

det ($\vec{A}^t$) =[$\vee(\vec{a}^T_{11} \wedge \vec{a}^T_{22})$, $\vee(\vec{a}^C_{11} \wedge \vec{a}^C_{22})$, $\wedge(\vec{a}^U_{11} \vee \vec{a}^U_{22})$, $\wedge(\vec{a}^F_{11} \vee \vec{a}^F_{22})$]∨ [$\vee(\vec{a}^T_{12} \wedge \vec{a}^T_{21})$, $\vee(\vec{a}^C_{12} \wedge \vec{a}^C_{21})$, $\wedge(\vec{a}^U_{12} \vee \vec{a}^U_{21})$, $\wedge(\vec{a}^F_{12} \vee \vec{a}^F_{21})$]

= det ($\vec{A}$) [since by Rearranging ($\vec{a}^T_{12} \wedge \vec{a}^T_{21}$) =($\vec{a}^T_{21} \wedge \vec{a}^T_{12}$)]

This can be proved for n order matrices where the permutation is induced by the rearrangement of each σ in S_n = $|\vec{A}|$.

4.6. Definition

The determinant ordering $\vec{A} \leq \vec{B}$ in membership value of FQNSM of order $m \times n$ *is defined as* $\vec{A} \leq \vec{B} \Leftrightarrow det(\vec{A}) \leq det(\vec{B})$ or $\vec{A} \leq \vec{B} \Leftrightarrow |\vec{A}| \leq |\vec{B}|$.

4.7. Proposition

The determinant ordering is not a partial ordering.

Proof:

- det $(\vec{A})$ = det$(\vec{B})$ for every FQNSM then $\vec{A} \leq \vec{B}$. Reflexivity is true.
- $\vec{A} \leq \vec{B} \Rightarrow |\vec{A}| \leq |\vec{B}|$ and $\vec{B} \leq \vec{A} \Rightarrow |\vec{B}| \leq |\vec{A}|$ then $det(\vec{A})$ =det$(\vec{B})$ but $\vec{A} \neq \vec{B}$. Anti symmetric is not true.
- $\vec{A} \leq \vec{B}$, $\vec{B} \leq \vec{C} \Rightarrow \vec{A} \leq \vec{C}$ for all $\vec{A}, \vec{B}, \vec{C}$ in FQNSM $\vec{A} \leq \vec{B} \Rightarrow |\vec{A}| \leq |\vec{B}|$ and $\vec{B} \leq \vec{C} \Rightarrow |\vec{B}| \leq |\vec{C}|$ and $\vec{A} \leq \vec{C} \Rightarrow |\vec{A}| \leq |\vec{C}|$.Transitivity is true.

Thus it does not satisfies 3 axioms so it is not a partial ordering.

4.8. An Application to Dimensionality Reduction using FQNSM

Object oriented FQNSM with respect to parameter is defined as

$$\overline{O_\iota} = \left[\sum_j \left\langle\frac{\vec{a}^T_{ij}}{|P|}\right\rangle, \sum_j \left\langle\frac{\vec{a}^C_{ij}}{|P|}\right\rangle, \sum_j \left\langle\frac{\vec{a}^U_{ij}}{|P|}\right\rangle, \sum_j \left\langle\frac{\vec{a}^F_{ij}}{|P|}\right\rangle,\right] \ \& \ j = 1,2 \ldots.. n\ ,\ i = 1,2 \ldots \ldots m$$

Where |.| denotes the cardinality of the system. Parameter oriented FQNSM with respect to object is defined as

International Research Journal on Advanced Engineering and Management
https://goldncloudpublications.com
https://doi.org/10.47392/IRJAEM.2024.501
e ISSN: 2584-2854
Volume: 02
Issue:11 November 2024
Page No: 3405-3410

$$\vec{P_j} = \left[\sum_j \left\langle\frac{\vec{a}_{ij}^T}{|e|}\right\rangle, \sum_j \left\langle\frac{\vec{a}_{ij}^C}{|e|}\right\rangle, \sum_j \left\langle\frac{\vec{a}_{ij}^U}{|e|}\right\rangle, \sum_j \left\langle\frac{\vec{a}_{ij}^F}{|e|}\right\rangle,\right] \ \& \ j = 1,2 \ldots..n \,,\ i = 1,2 \ldots\ldots m$$

4.8.1. Score Matrix

$$S(M)= [S_{ij}] = <\vec{a}_{ij}^T + \vec{a}_{ij}^C - \vec{a}_{ij}^U \,.\, \vec{a}_{ij}^F > \quad \forall \quad i \,\&\, j$$

4.8.2. Threshold Value

$$S(M)= <(\vec{a}_{ij}^T)^M + (\vec{a}_{ij}^C)^M - (\vec{a}_{ij}^U)^M \,.\, (\vec{a}_{ij}^F)^M > \quad \forall \quad i \,\&\, j$$

Then,

$$T= \left[\sum_{ij} \left\langle\frac{\vec{a}_{ij}^T}{|e\times P|}\right\rangle, \sum_{ij} \left\langle\frac{\vec{a}_{ij}^C}{|e\times P|}\right\rangle, \sum_{ij} \left\langle\frac{\vec{a}_{ij}^U}{|e\times P|}\right\rangle, \sum_{ij} \left\langle\frac{\vec{a}_{ij}^F}{|e\times P|}\right\rangle,\right] \ \& \ j = 1,2 \ldots..n \,,\ i = 1,2 \ldots\ldots m$$

4.8.3. Algorithm

1. Build the fuzzy Quadripartitioned neutrosophic soft matrix of $(F, \vec{X})$.
2. 2.Find the object oriented matrix for the object $\vec{O_i}$ and the parameter oriented matrix for the parameter $\vec{P_j}$. Next compute the score matrix using equation.
3. 3.Find the threshold element and threshold value of the FQNSM in proposed equation.
4. Remove those objects for which $S(\vec{O_i}) < S(T)$ and those parameters for which $S(\vec{P_j}) > S(T)$.
5. The new FQNSM with higher value is the desired dimensionality reduction.

Conclusion

Based on FQNSS, the current topic focuses on a novel kind of matrix theory called FQNSM theory. According to this new theory, decision makers should pay close attention to making accurate choices while dealing with uncertainty that involves indeterminacy. Indeterminacy can be broken down into two categories, namely ignorance and contradiction, which are more prevalent in the real world. Algebraic procedures on certain kinds of FQNSMs are also covered. In FQNSM, the accuracy and scoring functions are defined. To make actual decisions in the FQNSM environment, an algorithm has been devised. Finally, a challenge based on medical diagnosis has been satisfactorily resolved for the algorithm's actual use. The interval FQNSM theory, where the degrees of truth, contradiction, ignorance, and false membership are not clear, offers the researcher the opportunity to expand on the suggested issue in the future. Additionally, we use the suggested study in the areas of risk management, group DM problems, game theory, and similarity metrics.

References:

[1]. K.Atanassov,(1986) Intuitionistic fuzzy sets, *Fuzzy sets and systems*, 20 (1) ,87-96.

[2]. H.H.Cho, (1999),Regular fuzzy matrices and fuzzy equations, *Fuzzy sets and systems*, 105, 445-451.

[3]. Jian Mia Chen, (1982) Fuzzy Matrix partial ordering an generalised inverse, *Fuzzy sets systems*,1(5),453-458.

[4]. Molodtsov D, (1999) Soft set Theory first Results , *Computer and Mathematics with applications*, 37,19-31.

[5]. F.Smarandache,(2005) Neutrosophic set , A Generalisation of Intuitionistic Fuzzy sets, *International Journal of Pure and Applied Mathematics*,24, 287-297.

[6]. Maji.P.K,(2013) Neutrosophic soft set, *Annals of Fuzzy mathematics and Informatics*, 5, 157-168.

[7]. Somen Debnath, (2022) Fuzzy Quadripartitioned Neutrosophic Soft matrix theory and its Decision making, *Journal of computational and Coginitive Engineering*, 1(2), 88-93.

[8]. Kamal, (2021) Rough approximation of complex quadripartitioned single valued neutrosophic sets, *Journal of new theory*, 34, 45-63.

[9]. K. H. Kim and F. W. Roush, (1980) Generalised fuzzy matrices, *Fuzzy sets and systems*, 4 , 293-315.

[10]. K. H. Kim and A.Baartmans, (1989) Determinant theory for fuzzy matrices, Fuzzy sets and systems, 26,349-356.

[11]. Abishek Guleria and Rakesh Kumar Bajaj,(2019) Technique for reducing Dimensionality of data in decision making utilizing neutrosophic soft matrices, Neutrosophic sets and systems , University of Mexico, 29,129-141.

[12]. I.Arockiarani and I.R. Sumathi ,(2013) Fuzzy Neutrosophic soft topological spaces, IJMA,4 (10) , 2013,225-238.

[13]. I.R. Sumathi and I.Arockiarani,(2014) New

e ISSN: 2584-2854
Volume: 02
Issue:11 November 2024
Page No: 3405-3410

operations on Fuzzy Neutrosophic soft matrices, International Journal of Innovative research and studies, 3(3), 110-124.
[14]. Deli. I and Broumi.S,(2015) Neutrosophic soft matrices and NSM decision making, Journal of intelligent and fuzzy systems, 28, 2233-2241.
[15]. S.Khan and Anita Pal, (2007) The Generalised inverse of intuitionistic fuzzy matrices, Journal of Physical science, 11, 62-67.
[16]. R.Uma, P.Murugadas, and S. Sriram, (2017) The Determinant and adjoint of Fuzzy Neutrosophic Soft Matrices, International Journal of Mathematics and its Applications, 5(4-F),821-833.
[17]. M.Kavitha, P.Murugadas and S.Sriram , (2018) T- ordering on Fuzzy Neutrosophic Soft Matrices, International Journal of Pure and Applied Mathematics, 120(6), 893-907.
[18]. M.Kavitha, P.Murugadas and S.Sriram , (2018) Minus ordering on Fuzzy Neutrosophic Soft Matrices, International Journal of Pure and Applied Mathematics, 120(6), 1111-1121.
[19]. Chatterjee.R, Majumdar.P, and Samanta. (2016) S.K, On some similarity measures and entropy on quadripartitioned single valued neutrosophic set, Journal of Intelligent and fuzzy System, 30, 2475-2485.
[20]. Borah .M.J, Neog.T.J, and Sut.D. K, (2012) Fuzzy soft matrix theory and its decision making, International Journal of Modern Engineering and Research , 2,121-127
[21]. Kumar.S.R and Mary.S.A, (2021) Quadripartitioned Neutrosophic soft set, International Research journal on Advanced Science Hub, 3, 106-112.
[22]. R.Uma, P.Murugadas, and S. Sriram, (2021) Fuzzy Neutrosophic Soft Matrices of Type -I and Type-II , Fuzzy Information and Engineering,13, 211-222.
[23]. [Su.B, Ding.X , Wang.H and Wu .Y, (2018) Discriminative dimensionality reduction for multi –dimensional sequences, IEEE Transactions on Pattern analysis and Machine Intelligence, 40 (1), 77-91.
[24]. Xu.X, Liang.T, Zhu.J, Zheng.D, (2018) Dimensionality Reduction and sample selection methods for large scale data processing, Neutro computing, 2018.
[25]. Yong yang and Chenli ji, (2011) Fuzzy soft Matrices and their application”,LNAI, 7002,618-627.
[26]. A.R.Meenakshi, Fuzzy matrix theory and applications , MJP Publishers, Chennai-2008.