International Research Journal on Advanced Engineering and Management
https://goldncloudpublications.com
https://doi.org/10.47392/IRJAEM.2025.0023
e ISSN: 2584-2854
Volume: 03
Issue:02 February 2025
Page No: 130-136

Determination and Analysis of Domination Numbers for Boundary Graph and Boundary Neighbour Graph Using MATLAB

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Abstract
Vertex domination is a key concept in graph theory, essential for analyzing the structural properties of graphs. This study explores the use of vertex domination to determine the domination numbers for Two specific graph structures: boundary graphs and boundary Neighbor graphs. A graph's dominance number is the minimal collection of vertex in which each vertex is simultaneously in the prevailing set and Neighboring to one of its vertex. A boundary graph is created by adding a pendant border to a circle, while the boundary graph consists of k linearly arranged paths joined at a common vertex, known as the spine. This research aims to identify the larger for each of these structures and employs MATLAB to implement the theoretical algorithms computationally. A MATLAB-based algorithm is provided to compute the domination number γ(H), identify the dominating set, and visualize the graph with highlighted dominated vertices. Computer experiments are conducted to test and validate the theoretical findings. This paper demonstrates the practical application of domination numbers using MATLAB, offering insights into the efficiency and structural properties of these graphs in both theoretical and real-world contexts.
***Keywords:** Graph theory, Vertex domination, Domination number, MATLAB, Algorithm, Boundary Graphs and Boundary Neighbor Graphs, Graph design.*

1. Introduction
The ideas of dominating factors in the study of graphs is a commonly accepted topic in combinatorics are with a wide range of potential uses in fields such as information technology, networking sites, telecommuting, and structure design. A dominance set is an accumulation of vertices in a graph theory structure that is either Neighboring to the minimum of one vertex in the entire set or includes every vertex in the graph. The dominating number denotes the magnitude of the minimum dominating set in the graph. [1]. This research examines the domination numbers of specific graph types, including Boundary Graphs (BGs) and Neighboring Graphs $(BNGs)$, which are commonly used in fields like spatial and communication design. The objective is to establish a structured method for calculating and analyzing these domination numbers, utilizing MATLAB as a robust platform for numerical analysis and algorithm development[2]. minimum number of vertices necessary.

2. Graph Theory and Domination Number
In a graph $G = (V, E)$, where V indicates the collection of endpoints (or nodes) as well as E indicates the collection of connections (or connections), the biggest portion of D is defined to be the portion of V with the value that: $\forall\, u \epsilon\, V$, either $v \,\epsilon\, D$ or $\exists\, u \epsilon\, D$ such that $(u, v) \epsilon\, E$. The dominant amount of the graph G, designated as $\gamma(G)$, is the minimum number of vertices necessary to create a set that is dominant in the graph. In simple terms, it is the smallest number of a set D, which is a subset of each of the edges V of the data structure and in which every vertex throughout the graph has become

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either contained within D or Neighboring to at least a single intersection in D. [20]. This value provides insight into how productive and thorough a graph or framework is by determining the minimal number many nodes necessary to supervise or regulate the whole network[3].

3. Boundary Graph (BGs)

A Boundary Graph (BG) is a specialized class of graphs that models systems with spatial constraints or physical boundaries, often representing Graphs where nodes or edges are confined to specific regions or areas[4]. These Graphs are often seen in environmental modeling, communication Graphs, or sensor Graphs. In such Graphs, nodes are often situated at the boundary of a region, and their interconnections are constrained by the physical limits of the space they occupy. The dominating set in a boundary Graph not only ensures that every vertex is covered but also that the set of dominating nodes can be efficiently selected based on these boundary constraints [5]. A point v in $N(x)$ is considered a boundary point of u if the distance between p and v is shorter than the distance between p and x. If v is the nearest point to u, it is referred to as u's border Neighbor [6]. There is only one bordering network for a graph G if either v is an outer node of u in H or u is a barrier edge of v in H. This means that $Gb(G)$ has the same set of edges as H. Figure 1 shows Boundary Graph.

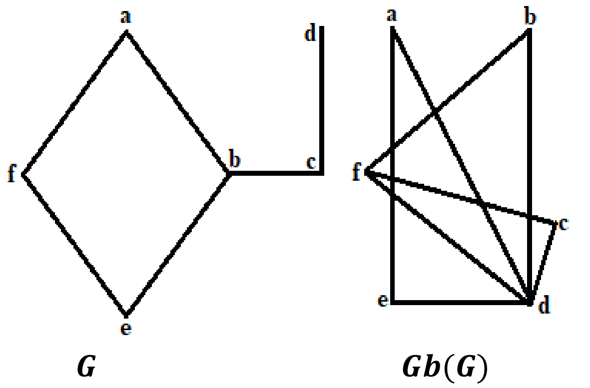

Figure 1 Boundary Graph

Every point in a chart has at least one border point, so a boundary graph doesn't have any single vertices. This is generally the case, however if G represents an unconnected plot, therefore $Gb(G)$ is likewise a graph that is unconnected. When $G = C4, Gb(G) = 2K$, 4 vertices arranged in a cycle), the boundary graph Gb(G) will be a graph that is isomorphic to 2K, which represents two disconnected vertices. This means that the boundary graph in this case is unconnected, with no edges between the vertices.[7]. Figure 2 shows Vertices

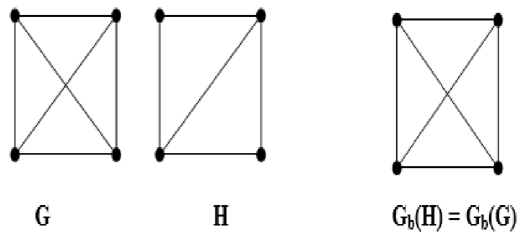

Figure 2 Vertices

A graph's eccentric graph G is always the border graph's subgraph $G_b(G)$.

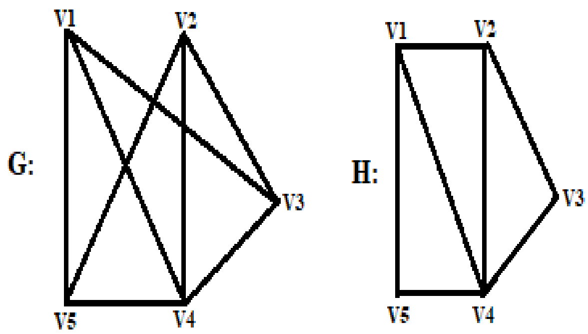

Figure 3 Boundary Graph G = G_b(H)

A graph, known as G, can be considered to be a boundary network whenever there existed a different graph H whose border is equal to G, represented by $Gb(H) = G$. This indicates that the arrangement of graph G may be determined by looking at the border or connections to the outside of network H [8]. Figure 3 shows Boundary Graph G = Gb(H)

3.1 Boundary Neighbour Graphs (BNGs)

A Boundary Neighbor Graph (BNG) is a variation of boundary Graphs where nodes are connected to their boundary Neighbors, typically adjacent vertices that lie on the boundary of the Graph [9]. The relationship between a node and its boundary Neighbors is critical for determining domination

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numbers, as the set of dominating vertices is impacted by the proximity and connectivity of boundary nodes. For a given vertex V, its boundary Neighbors are defined as the vertices that are adjacent to V and lie on the boundary of the Graph[10]. Mathematically, we can assume the set of boundary Neighbors of a vertex $u \epsilon V$ as:

$$N_{Boundary}\ (u) = \{u \in V : (u, v) \epsilon E\}$$

E and u is on the boundary of the Graph. Thus, domination in a BNN depends not only on the Graph's internal connectivity but also on the positioning of nodes relative to the boundary and their respective Neighbors[11].

3.2 Boundary Neighbors Graph

The border Neighbour graph of a given graph G, denoted as $BN(G)$, is a new graph constructed by retaining the same set of vertices as G. In this graph, two vertices u and v are adjacent if either u is a border Neighbor of v or vice versa in the original graph G,. A graph G, is considered 2-self-centered if it remains unchanged under certain transformations[12]. However, for some graphs, the border Neighbor graph $BN(G)$ may not be identical to G, even if G is 2-self-centered. For instance, if $G = C5 + e$, a 2-self-centered graph, then the border Neighbor graph $BN(G)$ will not be equal to G. Figure 4 shows Boundary Neighbours Graph.

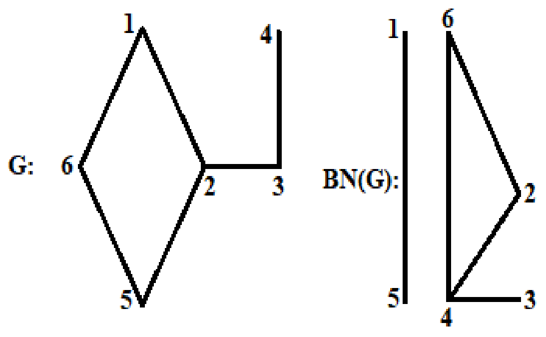

Figure 4 Boundary Neighbours Graph

$BN(G)$ is a sub graph of $Gb(G)$. If P = $v_1 v_2 \dots . v_P$ is a diametric route in G, then some of G's vertices have v_1 and v_P as border neighbors. G thus has a min. of 2 border neighbors [13]. In a boundary neighbors graph, each vertex will have at least one degree since each vertex in a linked network has at least one border neighbors. One kind of graph known as odd cycles is one for which $BN(G)$ □ G. If G is the complete chart K_p, then $BN(G) = G *$ $. Gb(G) = BN(G)$ if as well as only if for $v \epsilon V(G)$, All of v's border vertex are its boundary neighbors. $BN(G)$ can be either linked or detached. Additionally, there are no solitary vertex in the structure of $BN(G)$, since each vertex is linked to the minimum of one border vertex. Considering the pendant crossover in G, represented by vertex v. If v is a pendants, it is next to other the vertex and u, in the data structure[14]. Consider uv be a component of $BN(G)$, that corresponds to the graph's border vertices. Thus, $BN(G)$ represents an element of G. A boundaries vertex in G is called a pendant vertices if it only occurs once there exists an individual vertex v in G with precisely one neighbors in $G's$ border, meaning that vertex v cannot be used as an outside neighbors for each additional vertex within the group. Furthermore, consider G to be a graph with a radius of one [15]. The main focus of this is to determine and analyze the domination numbers of Boundary Graphs and Boundary Neighbor Graphs. The problem involves calculating the smallest dominating set for each type of Graph and comparing their characteristics in terms of connectivity, efficiency, and optimization [16]. To compute the domination number for these Graphs, we will use MATLAB, which provides a variety of functions for matrix operations, graph theory algorithms, and visualization tools. The MATLAB environment will be used to simulate boundary Graphs and boundary Neighbor Graphs, implement algorithms for domination number calculation, and analyze the results.

4. Boundary Graph

The dominant factor of a graph was the least number vertex that needs to be selected to ensure that each vertices within the structure either appears in the group you have selected or has a connecting edge linking it to a vertex in the group of choice [17]. The concept of control numbers is at the core of graph theory and has applications in many fields, including computer networks, social networks. and resource allocation problems.

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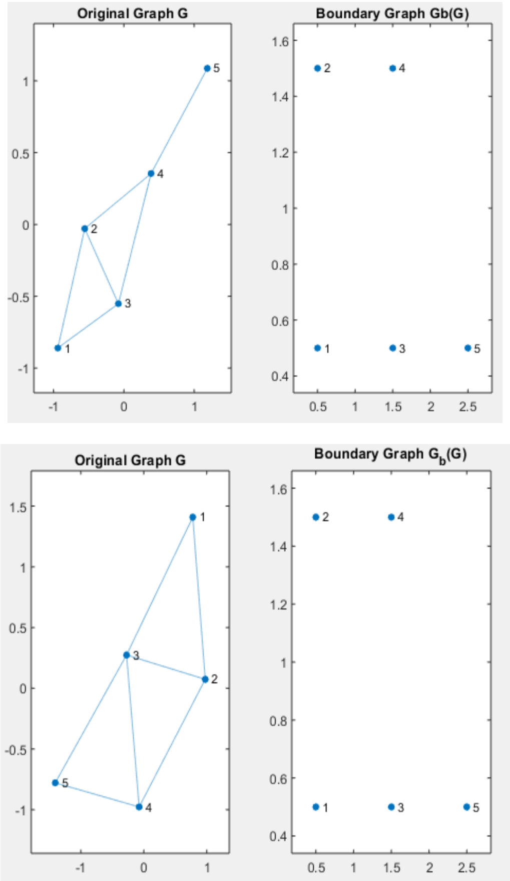

Figure 5 Boundary Graph G_b (G)

In graph theory, boundary graphs are a fundamental idea, especially when it comes to the use of vertex dominance. A Neighbors Boundary Graph One kind of boundary graph that emphasises the connection between dominant sets and boundary vertices is called $BN(G)$. The importance of boundary vertex in domination theory can be demonstrated by the assertion that each node in the dominant collection in $BN(G)$ has a minimum of some non-adjacent graph that forms its boundaries[18]. comprehension the structural characteristics of dominant sets and how they affect graph resilience and connection requires a comprehension of this graph. the domination numbers of Boundary Graphs and Boundary Neighbor Graphs. The problem involves calculating the smallest dominating set for each type of Graph and comparing Vertex dominance in graph theories is theoretically supported in large part by the research of $BN(G)$. Figure 5 shows Boundary Graph Gb (G)

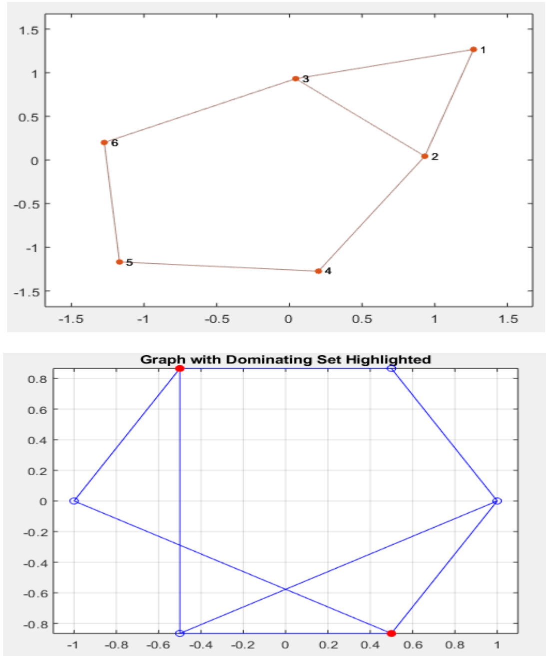

Figure 6 Graph with Dominating Set

In graph theory, a boundary graph is often used to represent the connections and relationships between vertices in a network or system [19]. The matrix of adjacency within the program describes the relationships connecting six vertices, with 1 indicating a boundary connecting both vertices and 0 indicating no link. A domineering set in a data structure is a collection of triangles which means each additional vertex in the structure is either a member of the collection or linked to no less than a single vertex in the collection via an edge. In this situation, the biggest integer in the dominant set is 2. The set of numbers comprises vertices between two and five[20]. These two vertices ensure that all others are covered, either directly or indirectly through an edge. Figure 6 shows Graph with Dominating Set.

5. Boundary Neighbours Graph

Boundary Neighbour Graph BN(G) is a graph theoretical concept that represents a graph G where each vertex is replaced by its boundary Neighbors. This graph is formed by replacing each vertex with its boundary Neighbors and forming edges between these Neighbors. BN(G) is instrumental in various graph theoretical applications, particularly in understanding vertex domination and network

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properties [21]. It is an important part of the field of graph theory since it examines the arrangement inherent interconnection of graphical representations, as well as several other features. Its applications extend across multiple fields, including computer science, social networks, and biological networks. Figure 7 shows Boundary Neighbours Graph BN(G) Figure 8 shows Boundary Neighbours Graph.

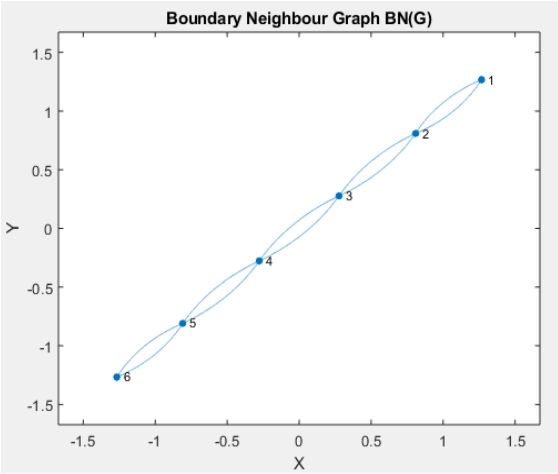

Figure 7 **Boundary Neighbours Graph $BN(G)$**

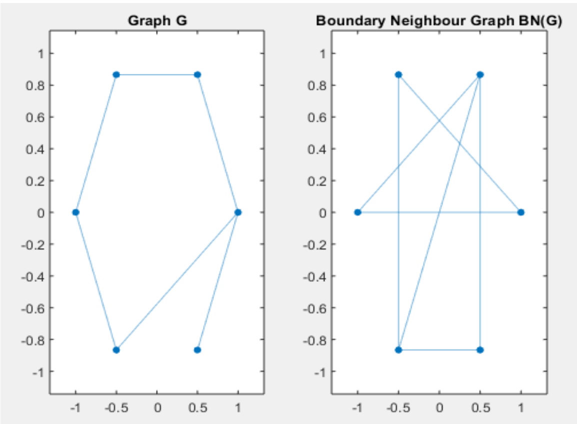

Figure 8 **Boundary Neighbours Graph**

The Boundary Neighbors Graph $BN(G)$ is constructed from an original graph G, which in this example is a path graph P6 with 6 vertices. The boundary neighbors of each vertex are found using a function that identifies adjacent vertices [22]. $BN(G)$ is then created by connecting each vertex to its boundary neighbors. The visualization below shows the original graph G and its corresponding Boundary Neighbors Graph BN(G).

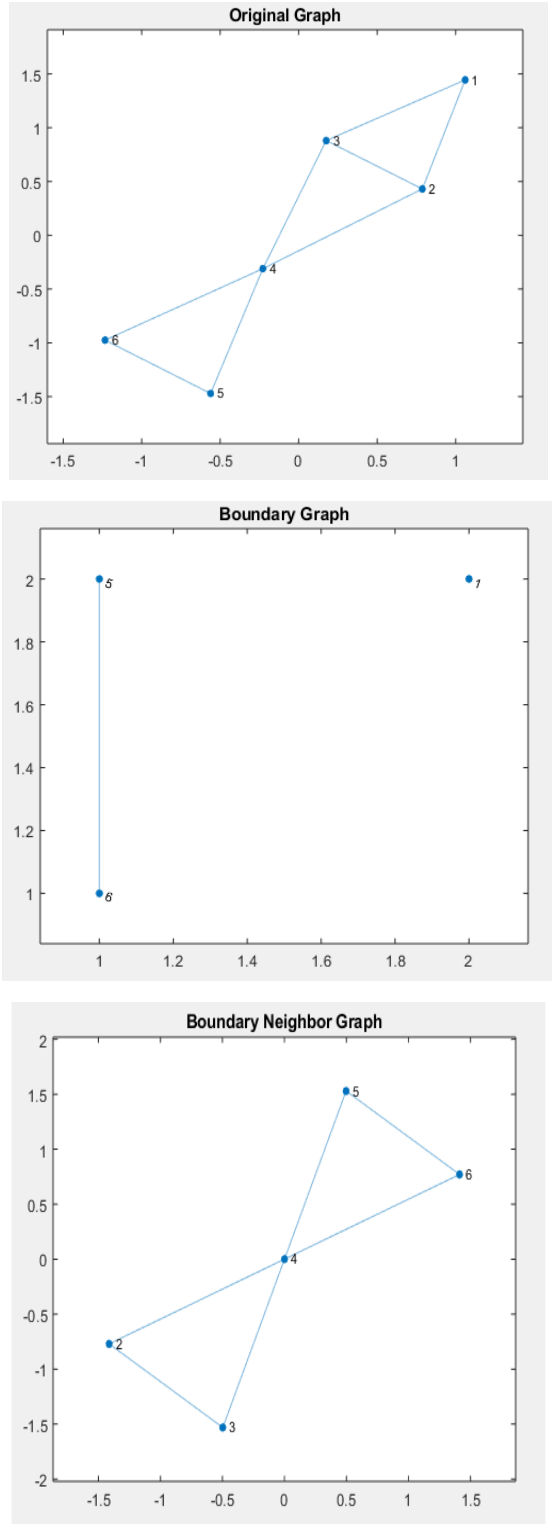

Figure 9 **Boundary Neighbours Graph**

The Boundary Graph represents a subgraph consisting of nodes with a degree of 1 or 2, indicating the boundary nodes of the original graph. In this case

[23], the boundary nodes are nodes 1, 5, and 6, as their degrees are either 1 or 2. The Boundary Neighbor Graph, on the other hand, is formed by including the Neighbors of these boundary nodes [24]. This graph highlights the connections between boundary nodes and their adjacent non-boundary nodes. In this case, the boundary Neighbor nodes are 2, 3, 4, 5, and 6, reflecting the Neighboring relationships between boundary and non-boundary nodes [25]. Figure 9 shows Boundary Neighbours Graph.

Conclusion

This study examines the determination and analysis of domination numbers in Boundary Graphs (BGs) and Boundary Neighbor Graphs (BNGs) using MATLAB, a powerful tool for graph theory analysis. Domination numbers, which represent the least number of vertices in a dominating set required to cover all vertices in a graph, play a critical role in optimizing network structures and ensuring efficient monitoring and control in systems such as telecommunications, sensor networks, and social networks. The analysis focused on two types of graphs: BGs, which model systems with spatial constraints or physical boundaries, and BNGs, which emphasize the relationships between boundary vertices and their Neighbours. The MATLAB-based approach developed in this research allowed for the systematic calculation of domination numbers, revealing that the domination number for a BG can be significantly influenced by boundary constraints, while the BNG often exhibits increased complexity due to the connectivity between boundary and non-boundary vertices. Through visualization and algorithmic implementation, the study demonstrated that the domination number provides valuable insights into the efficiency, resilience, and coverage of a graph. Ultimately, this work highlights the importance of boundary and Neighbor relationships in graph structures, offering a deeper understanding of domination numbers in spatial and communication networks.

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