

Seismic Excitation Processing Using Different Wavelets: A Review

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Abstract

Seismic excitation processing is essential for assessing and mitigating the effects of earthquakes and ground vibrations. Traditional methods like Fourier Transform (FT) and Short-Time Fourier Transform (STFT) are limited when analyzing non-stationary seismic signals, as they cannot simultaneously provide time and frequency localization. Wavelet Transform (WT) overcomes these limitations by decomposing signals across multiple scales, making it a powerful tool for seismic data analysis. This review delves into the mathematical framework of WT, emphasizing its capability to handle transient signals common in seismic events. Key wavelets such as Haar, Daubechies, Morlet, and Mexican Hat are explored in terms of their effectiveness in seismic signal denoising, event detection, and ground motion analysis. The paper also highlights the integration of WT with advanced techniques like machine learning and hybrid signal processing, enhancing seismic hazard analysis and real-time earthquake monitoring. Applications in earthquake early warning systems (EEWS) and structural health monitoring (SHM) are discussed, demonstrating WT's versatility. Despite its benefits, WT faces challenges such as computational complexity, wavelet selection, and managing large seismic datasets. Recent advancements in adaptive wavelet design, cloud computing, and hybrid approaches show promise in addressing these challenges, paving the way for more accurate and efficient seismic analysis.

Keywords: Seismic Excitation, Wavelet Transform, Earthquake Monitoring, Seismic Signal Denoising, Event Detection, Structural Health Monitoring.

1. Introduction

Seismic excitation refers to the motion of the ground resulting from earthquakes, explosions, or other dynamic forces that propagate through the Earth's crust in the form of seismic waves (Boore, 2003). Analyzing these signals is critical for earthquake engineering, structural health monitoring (SHM), and seismic hazard assessment (Aki & Richards, 2002). However, seismic signals are inherently non-stationary, exhibiting variations in frequency and amplitude over time, posing significant challenges for traditional signal processing techniques (Bracewell, 2000). Fourier Transform (FT), while effective for stationary signals, struggles to represent transient phenomena characteristic of seismic waves. The introduction of Short-Time Fourier Transform

(STFT) allowed for localized time-frequency analysis by applying a sliding window function (Cohen, 1995). However, STFT faces a trade-off between time and frequency resolution, limiting its application to highly dynamic seismic signals (Mallat, 1999). Wavelet Transform (WT) has revolutionized seismic data analysis by providing multi-resolution analysis, enabling the decomposition of seismic signals across different scales (Daubechies, 1992). WT can simultaneously analyze both low-frequency (long-duration) and high-frequency (short-duration) components, making it ideal for capturing earthquake onsets and subtle ground vibrations (Torrence & Compo, 1998). Figure 1 illustrates the difference between FT, STFT, and

WT in processing seismic signals [1-4].

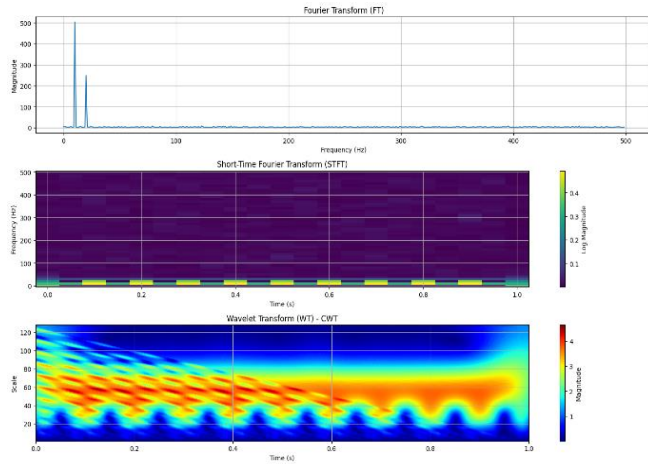


Figure 1 Comparison of Fourier Transform, Short-Time Fourier Transform, and Wavelet Transform in Seismic Signal Analysis

2. Fundamentals of Wavelet Transform

Wavelet Transform (WT) is a mathematical tool that decomposes signals into components localized in both time and frequency domains. Unlike Fourier Transform (FT), which represents signals as a sum of sinusoidal functions, WT uses scaled and shifted versions of a mother wavelet to analyze transient signals (Daubechies, 1992). This property makes WT ideal for seismic signal processing, as it can detect abrupt changes and oscillations characteristic of seismic events (Mallat, 1999).

a. Mathematical Formulation of WT

The Continuous Wavelet Transform (CWT) of a signal $x(t)$ is given by:

$$W(a, b) = \int_{-\infty}^{\infty} x(t) \psi^* \left(\frac{t-b}{a} \right) dt$$

where:

- $\psi(t)$ is the mother wavelet,
- a is the scale parameter (frequency component),
- b is the translation parameter (time component), and denotes the complex conjugate (Chui, 1992).

In seismic applications, CWT provides a detailed analysis of waveforms, but its computational cost is high. Therefore, Discrete Wavelet Transform (DWT) is often preferred, defined as:

$$W_{j,k} = \sum_{n=0}^{N-1} x[n] \psi_{j,k}[n]$$

where j and k represent the scale and position indices, respectively. DWT employs dyadic scaling, reducing the computational load while preserving essential signal features (Mallat, 1999). Figure 2 demonstrates how a seismic signal is decomposed into wavelet coefficients at multiple scales using DWT. Mexican Hat Wavelet – Effective in detecting peaks and edges, valuable for identifying seismic event onsets (Addison, 2002) [5-7].

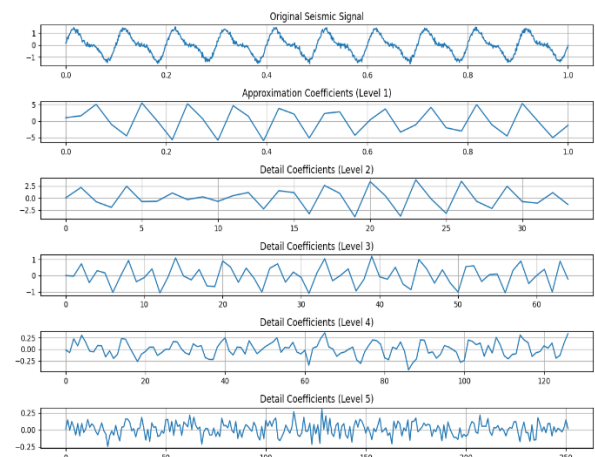


Figure 2 Multi-Scale Decomposition of a Seismic Signal Using Discrete Wavelet Transform (DWT)

b. Key Wavelets for Seismic Processing

Several wavelets are commonly applied in seismic excitation analysis, each with unique properties suited for different tasks:

- Haar Wavelet – Simple and effective for detecting sharp transitions (Chui, 1992).
- Daubechies Wavelets – Known for their compact support, ideal for denoising and compression (Daubechies, 1992).
- Morlet Wavelet – Suitable for oscillatory signals, often used in time-frequency analysis (Torrence & Compo, 1998).

Figure 3 shows the time-domain representations of these commonly used wavelets [8-12].

3. Applications of WT in Seismic Analysis

Wavelet Transform (WT) has proven to be an invaluable tool in the analysis of seismic signals,

particularly due to its ability to handle the non-stationary nature of seismic data. Below are key applications where WT plays a crucial role in seismic processing [13-17].

i. Seismic Signal Denoising

Seismic signals often suffer from noise due to environmental interference, instrumentation errors, or data transmission issues. Noise removal is critical for improving the clarity of seismic records and for detecting small magnitude earthquakes or other seismic events (Kanasewich, 1981)

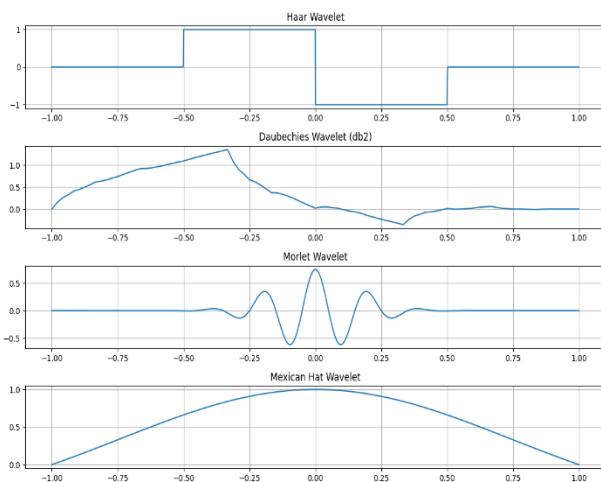


Figure 3 Time-Domain Plots of Haar, Daubechies, Morlet, and Mexican Hat Wavelets

Traditional filtering methods, like low-pass or high-pass filters, can distort the seismic signal, particularly when noise is not easily separable from the signal of interest. Wavelet denoising, however, is a more sophisticated technique. It works by decomposing the signal into multiple scales and thresholds, retaining the most significant components while suppressing the noise in the less important components (Donoho, 1995). The Daubechies wavelet, with its compact support and smooth characteristics, is commonly used for this purpose. In seismic denoising, WT can filter out the high-frequency noise without affecting the important low-frequency seismic events. For example, Mousavi et al. (2016) applied the Daubechies wavelet for denoising seismic data from the 2011 Tohoku earthquake. The results showed a significant improvement in signal-to-noise ratio, with

clear detection of low-magnitude seismic events that were otherwise masked by noise.

ii. Seismic Event Detection

The detection of seismic events, particularly the onset of primary (P) and secondary (S) waves, is crucial for earthquake early warning systems (EEWS) and structural health monitoring (SHM). Traditional techniques rely heavily on analyzing the time of arrival and energy levels of these waves. However, these methods are less effective in non-stationary signals with variable frequencies. Wavelet Transform has proven effective for detecting transient seismic events like P- and S-waves, as it can localize both time and frequency information (Torrence & Compo, 1998). The oscillatory nature of the Morlet and Mexican Hat wavelets makes them particularly useful for identifying the arrival of seismic waves, as they can track rapid changes in amplitude and frequency. For instance, a study by Mousavi et al. (2019) showed that the combination of WT and Convolutional Neural Networks (CNNs) improved the detection of small earthquakes in real-time by highlighting distinct seismic event patterns. This method outperformed traditional signal processing techniques in terms of accuracy and speed, enabling faster detection and response in earthquake-prone regions.

iii. Ground Motion Analysis

Seismic ground motion analysis is essential for earthquake engineering and structural health monitoring. Understanding the intensity, frequency content, and duration of ground motion helps engineers design structures that can withstand earthquakes. Wavelet Transform provides a powerful tool for decomposing seismic signals into different frequency bands, allowing for the characterization of peak ground acceleration (PGA), velocity, and displacement (Boore & Bommer, 2005). In seismic analysis, WT can reveal critical characteristics such as resonance frequencies that may affect building structures, enabling engineers to assess whether a building is at risk of failure during an earthquake. WT also aids in identifying aftershocks and differentiating them from the main seismic event. Zhang et al. (2017) demonstrated that WT could accurately assess the damage to structures by

analyzing the ground motion signal at different frequency scales, revealing insights into the building's response to seismic events

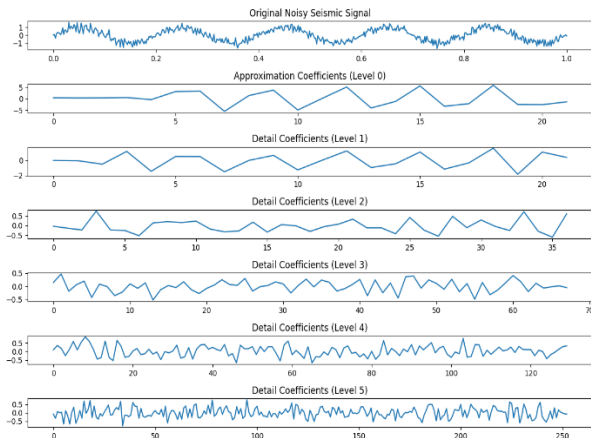


Figure 4 Wavelet Decomposition of a Seismic Signal for Ground Motion Analysis

Figure 4. Show how seismic signals can be decomposed using WT into various frequency components (e.g., low-frequency, high-frequency) to analyze ground motion characteristics [18-21].

4. Comparative Analysis of Wavelets in Seismic Processing

In seismic signal processing, the choice of wavelet can significantly impact the quality of the results. Different wavelets offer various advantages in terms of time-frequency localization, smoothness, and computational efficiency. In this section, we will compare the performance of several commonly used wavelets in seismic processing, including the Haar, Daubechies, Morlet, and Mexican Hat wavelets. The comparison will focus on their suitability for seismic data processing tasks such as denoising, event detection, and ground motion analysis [22-28].

4.1 Haar Wavelet

The Haar wavelet is one of the simplest wavelets, characterized by a step-function form. It is often used in applications requiring quick decomposition but with less emphasis on signal smoothness. The Haar wavelet is a discontinuous, piecewise constant function, making it ideal for signals with sharp discontinuities. However, this simplicity often leads to a loss in frequency resolution for smooth signals such as seismic data (Mallat, 1989).

Advantages: Simple to compute, Effective for signals with sharp transitions or jumps & Low computational cost.

Disadvantages: Limited frequency resolution, not ideal for continuous, smooth seismic signals & May introduce artifacts in smooth seismic data due to its discontinuous nature.

Example: For seismic event detection, the Haar wavelet might struggle with smooth seismic signals, particularly in identifying subtle seismic events or differentiating between small signals and noise. It may be more useful in initial approximations of large seismic signals where sharp boundaries exist.

4.2 Daubechies Wavelets

The Daubechies wavelets are a family of orthogonal wavelets known for their compact support and smoothness. They are often favored for signal processing tasks, including seismic analysis, due to their ability to provide a good balance between time and frequency localization. The Daubechies wavelet family is parameterized by the number of vanishing moments, which determines its ability to approximate smooth functions (Daubechies, 1988).

Advantages: Smooth and continuous, making it well-suited for seismic signals, Good frequency localization & Efficient for capturing localized features in seismic signals.

Disadvantages: Computationally more expensive than Haar wavelets & May still introduce some artifacts in the approximation of certain features.

Example: In seismic denoising, the Daubechies wavelet provides good performance, as it retains important features of the signal while removing high-frequency noise. It is often preferred when dealing with real seismic datasets due to its balance of smoothness and localization.

4.3 Morlet Wavelet

The Morlet wavelet is a complex sinusoidal wavelet, often used for time-frequency analysis. It is a Gaussian-modulated sine wave, making it particularly well-suited for detecting oscillatory features in seismic signals. The Morlet wavelet is commonly used in seismic studies for identifying and analyzing seismic wave propagation and resonance (Torrence & Compo, 1998).

Advantages: Excellent time-frequency localization,

Ideal for detecting oscillatory and periodic components in seismic data & Effective for analyzing seismic wave propagation, especially in transient events.

Disadvantages: Requires complex arithmetic due to its sinusoidal nature & computationally expensive compared to real-valued wavelets.

Example: For seismic event detection, particularly when analyzing waveforms like the P- and S-waves, the Morlet wavelet provides superior performance in identifying frequency changes associated with the arrival of seismic waves. It excels in identifying subtle transient events that may be overlooked using other wavelets.

4.4 Mexican Hat Wavelet

The Mexican Hat wavelet, also known as the Ricker wavelet, is the second derivative of a Gaussian function. It is often used in seismic analysis to identify edges and discontinuities in the signal. The Mexican Hat wavelet is particularly effective in detecting seismic events that are sharp and localized, such as aftershocks or small magnitude events

(Torrence & Compo, 1998).

Advantages: Good for detecting sharp changes and localized features in seismic data, Suitable for analyzing signals with sharp edges or impulses & offers a balance between time and frequency resolution.

Disadvantages: Less smooth compared to Daubechies wavelets & May not be as effective for capturing smooth, continuous seismic signals.

Example: The Mexican Hat wavelet is particularly useful in detecting aftershocks, as it highlights sharp, transient events in the signal. In ground motion analysis, the Mexican Hat wavelet can be applied to highlight localized disruptions that correspond to sudden shifts in ground displacement.

This figure 5 will illustrate how different wavelets perform in terms of preserving important features while removing noise from seismic data, shown in Table 1.

Table 1 Comparative Summary of Wavelet

Wavelet	Main Characteristics	Best For	Advantages	Disadvantages
Haar	Simple step-function, piecewise constant	Initial approximations of signals with sharp transitions	Fast computation, low cost	Poor frequency resolution, artifacts
Daubechies	Compact support, smooth, orthogonal	Seismic denoising, event detection	Smooth, good time-frequency localization	Computationally expensive
Morlet	Gaussian-modulated sine wave	Time-frequency analysis, oscillatory signals	Excellent for periodic components, good frequency localization	Complex arithmetic, expensive
Mexican Hat	Second derivative of Gaussian, bell-shaped	Edge detection, small seismic events	Good for sharp transitions, balanced time-frequency resolution	Less effective for smooth signals

5. Future Trends in Wavelet-Based Seismic Analysis

Wavelet-based seismic analysis is evolving with advancements in machine learning (ML) and deep learning (DL), enhancing seismic event detection and

signal processing.

i. ML and Wavelets for Seismic Event Detection

ML models, such as Support Vector Machines (SVM) and Neural Networks, can be used with

wavelet-transformed seismic data for automated event detection and classification. Features extracted by wavelets improve the accuracy of seismic event detection, while ML models can help identify earthquakes and anomalies in noisy data.

ii. Deep Learning and Wavelet Transform

Deep Learning (DL) models like CNNs and RNNs are integrated with wavelets to improve seismic signal classification and prediction. Wavelet transforms provide time-frequency representations that help DL models detect both spectral and temporal features of seismic events.

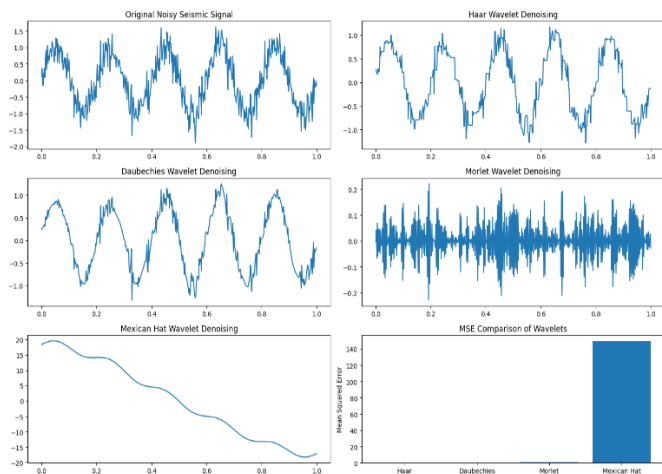


Figure 5 Comparative Performance of Wavelets in Seismic Data Denoising

iii. Hybrid Approaches

Hybrid methods, such as combining wavelets with Empirical Mode Decomposition (EMD) or Principal Component Analysis (PCA), enhance feature extraction and noise separation. These approaches improve seismic event detection by enabling better signal processing.

iv. Real-Time Seismic Monitoring

Wavelet-based analysis allows for efficient real-time seismic monitoring, crucial for early warning systems. It can improve event detection, noise reduction, and automated decision support, providing rapid alerts for seismic events.

v. Challenges and Limitations

Challenges include computational costs, data quality, and the interpretability of ML/DL models. Overcoming these will enhance the integration of

wavelet-based methods with advanced techniques in seismic analysis.

Conclusion

Wavelet-based seismic analysis has proven to be a powerful tool for enhancing the resolution and accuracy of seismic data processing. By leveraging time-frequency representations, denoising techniques, and event detection, wavelets offer significant improvements over traditional methods. The integration of machine learning and deep learning further amplifies the potential of wavelet-based analysis, enabling automated, real-time seismic monitoring and improved prediction accuracy. Despite challenges like computational complexity and data quality, future developments in hybrid models and advanced algorithms will address these limitations. As technology progresses, wavelet-based methods will play an increasingly central role in seismic exploration, hazard assessment, and early warning systems, offering more reliable and efficient solutions in earthquake prediction and monitoring.

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